Name: Enrolment No:	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2025

Course: Mathematical Science – 2 **Semester: II**

Program: B.Sc. CS **Time : 03 hrs.**

Course Code: MATH 1067 **Max. Marks: 100**

Instructions: Answer all questions. There is an internal choice in Q9 and Q11.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Using bisection method, find a root, correct to three decimal places and lying between 1 and 2 of the equation $f(x) = x^3 - x - 1 = 0$.	4	CO1
Q 2	Find the approximate value of the integral $\int_1^2 \frac{1}{5+3x} dx$, using Simpson's $-\frac{1}{3}$ rule with 4 equal subintervals.	4	CO1
Q 3	Define the singularity of a function. Determine and classify the singularities of $f(z) = \frac{e^{3z}}{z^2 sinz}$.	4	CO2
Q 4	Find the Laplace transform of the function $f(t) = (t+1)^2 e^{5t}$, using first shifting theorem.	4	CO3
Q 5	Evaluate $L^{-1}\left\{\frac{1}{s^2-5s+6}\right\}$.	4	CO3

SECTION B

(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Solve the following initial value problem and find y(0.2) using Ruge-Kutta method of fourth order: $\frac{dy}{dx} = 1 + y^2;\ y(0) = 0$ with $h = 0.2$.	10	CO1
Q 7	Find Laurent series about the indicated singularity for each of the following function: (a) $\frac{e^{2z}}{(z-1)^3}; z = 1$. (b) $\frac{z - sinz}{z^3}; z = 0$. Also, name the singularity in each case.	10	CO2

Q 8	Classify the following partial differential equations: (a) $2\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = 2.$ (b) Determine the set $S \subseteq R^2$ such that the partial differential equation $(x-1)^2 u_{xx} - (y-2)^2\, u_{yy} = 0$ is parabolic in S.	**10**	**CO4**
Q 9	Find the Fourier series of the function $f(x) = x^2$ in the interval $(0, 2\pi)$ and hence, deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \cdots$ **OR** (a) Express the following function in terms of unit step function and find it's Laplace transform: $f(t) = \begin{cases} 8, & t < 2 \\ 6, & t > 2. \end{cases}$ (b) Find the Laplace transform of $f(t) = \begin{cases} \frac{t}{k}, & 0 < t < k \\ 1, & t > k. \end{cases}$	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	A voltage Ee^{-t} is applied at $t = 0$ to a circuit of inductance L and resistance R. The equation governing the current flow in LR circuit is given by $L\frac{dI}{dt} + RI = Ee^{-t},\ I(0) = 0.$ Using Laplace transformation, find the current I at any time t.	**20**	**CO3**
Q 11	Derive the most general solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial x^2}$. Deduce the expression for $u(x,t)$ satisfying the boundary conditions $u(0,t) = 0 = u(l,t)$. **OR** Use the method of separation of variables to solve the following one-dimensional heat equation: $$\frac{\partial^2 v}{dx^2} = \frac{\partial v}{\partial t}$$ Given that $v = 0$ when $t \to \infty$, as well as $v = 0$ at $x = 0$ and $x = l$.	**20**	**CO4**