Name: **Enrolment No:**	

UPES

End Semester Examination, May, 2024

Course: Number Theory | **Semester: 6th**
Program: Int. B.Sc. - M.Sc. Mathematics | **Time : 03 hrs.**
Course Code: MATH4006P | **Max. Marks : 100**

Instructions: Read all the below-mentioned instructions carefully and follow them strictly:
1) Mention Roll No. at the top of the question paper.
2) ATTEMPT ALL THE PARTS OF A QUESTION AT ONE PLACE ONLY.

SECTION A
(5Qx4M=20Marks)

S. No.	Answer all the questions	Marks	CO
Q 1	Find all possible solutions of the equation $x^2 \equiv 8\ (mod\ 17)$.	**4**	**CO3**
Q 2	Let a, b, c be integers. If a divides bc and $(a, b) = 1$, then prove that a divides c.	**4**	**CO1**
Q 3	For any odd prime p, prove that the number of quadratic residues and quadratic non-residues are equal in number under modulo p.	**4**	**CO3**
Q 4	Find the value of $\left(\frac{13}{101}\right)$, notation has its usual meaning.	**4**	**CO3**
Q 5	Evaluate the value of $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60}\ (mod\ 7)$.	**4**	**CO2**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Find the order of 2 and 8 in $\mathbb{Z}_{27}$, and state which one is the primitive root. Find the list of all elements that are primitive in $\mathbb{Z}_{27}$.	**10**	**CO3**
Q 7	State Fermat's Litle Theorem. Prove that, for any n = pq with distinct primes p and q with a is not divisible by p or by q, then $a^{(p-1)(q-1)} \equiv 1\ (mod\ n)$. Hence evaluate $45^{6468}\ (mod\ 6499)$.	**10**	**CO2**
Q 8	Find all positive integers x such that $2^{2^x} + 2$ is divisible by 17.	**10**	**CO2**
Q9	If u and v are relatively prime positive integers whose product uv is a perfect square. Then u and v are both perfect squares. OR	**10**	**CO4**

	For any arithmetic function $f(n)$, is multiplicative iff its sum-function $S_f(n)$ is multiplicative, where $S_f(n)$ denote the sum of all the values of $f(n)$ at different n.		
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Let p and q are distinct odd primes of the form $4k+3$, then one of the congruences $x^2 \equiv p \ (mod\ q)$ and $x^2 \equiv q \ (mod\ p)$ is solvable and other is not. But if one of the prime is of the form $4k+1$, then prove that both the congruences are solvable or both are not.	**20**	**CO4**
Q 11	Let a and b be positive integers. If $(a,b) = 1$, then the number of positive integer n that cannot be written in the form of $ar + bs = n$, for some non-negative integers r, s equals $(a-1)(b-1)/2$. OR Let p be an odd prime, then prove that $\left(\frac{2}{p}\right) = (-1)^{(p^2-1)/8}$.	**20**	**CO3**