Name: **Enrolment No:**	

UPES

End Semester Examination, May 2024

Course: Computational Mathematics **Semester: VI**
Program: Integrated B.Sc.-M.Sc. Mathematics **Time: 03 hrs.**
Course Code: MATH 3049 **Max. Marks: 100**
No. of Pages: 03
Instructions: Answer all the questions.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Using Modified Euler's method, obtain a solution of the equation $\frac{dy}{dx} = x + \lvert\sqrt{y}\rvert$ with initial condition $y(0) = 1$ for the range $0 \le x \le 0.4$ in steps of 0.2.	**4**	**CO1**
Q 2	(a) Determine whether the following equation is elliptic or hyperbolic. $$(x+1)u_{xx} - 2(x+2)u_{xy} + (x+3)u_{yy} = 0.$$ (b) In which parts of the (x, y) plane is the following equation elliptic? $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + (x^2 + 4y^2)\frac{\partial^2 u}{\partial y^2} = 2\sin(xy).$$	**4**	**CO2**
Q 3	Define critical point and discuss the nature of the critical point of the following linear autonomous system: $$\frac{dx}{dt} = -5x + y; \frac{dy}{dt} = x - 5y.$$	**4**	**CO3**
Q 4	Discuss the classification of Mathematical Models based on their nature. Also classify the following models: (i) $x(t+1) = ax(t) - bx(t)y(t)$ $y(t+1) = -py(t) + qx(t)y(t)$ where $x(t)$ and $y(t)$ represent the populations of prey and predator species respectively. (ii) $\frac{dP_n}{dt} = \alpha P_{n-1}(t) - \beta P_{n+1}(t) - (\alpha+\beta)P_n(t); n = 1,2,3..$ and $P_n(t)$ is the probability of n persons at time t.	**4**	**CO4**
Q 5	Discuss the Linear Congruential method (LCM) for random number generation and using LCM, generate a sequence of 5 random numbers with $x_0 = 27, a = 17, c = 43$ and $m = 100$.	**4**	**CO5**

SECTION B **(4Qx10M= 40 Marks)**			
Q 6	Solve the equation $y'' - x^2y' - 2xy = 1, y(0) = 1, y'(0) = 0$ to obtain $y'(0.1)$ using Runge-Kutta method of order 4.	**10**	**CO1**
Q 7	Define Lyapunov's function and discuss Lyapunov's first method. Using the Lyapunov's first method, investigate the stability of the following system of equations: $\dot{x_1} = -3x_1 + x_2$ $\dot{x_2} = -x_1 - x_2 - {x_2}^3.$	**10**	**CO3**
Q 8	Discuss population growth and decay models. A colony of bacteria grows according to the law of uninhibited growth $P(t) = 100\, e^{0.045t}$ where P is measured in grams and t in days. Determine (a) The initial number of bacteria. (b) The growth rate of the bacteria. (c) The population after 5 days. (d) The time taken for the population to reach 140 grams. (e) The doubling time for the population.	**10**	**CO4**
Q 9	Solve $\nabla^2 u = 0$ under the conditions $u(0, y) = 0$, $u(4, y) = 12 + y$, for $0 \leq y \leq 4$; $u(x, 0) = 3x, u(x, 4) = x^2$ for $0 \leq x \leq 4$ by taking $h = k = 1$. Perform one iteration of Liebmann's method by obtaining initial approximations using standard (or diagonal) five-point formulae. **OR** Solve $u_{xx} - 16u_t = 0$ under the given conditions $u(x, 0) = 0$, $u(0, t) = 0$ and $u(1, t) = 200t$. Compute u for one time step with $h = 0.25$.	**10**	**CO2**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Discuss sign definiteness of scalar functions, matrices, and quadratic forms. Using Lyapunov's direct method, discuss the stability of the system $\dot{x} = Ax$, where $A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$. Also find the corresponding Lyapunov function.	**20**	**CO3**
Q 11	Discuss the Monte Carlo Simulation Technique and write the Monte Carlo algorithm to find the area enclosed by the curve $y = f(x)$ satisfying $0 \leq f(x) \leq M$ over the closed interval $a \leq x \leq b$ where M is a constant that bounds the function. **OR** A tourist car operator finds that during the past few months, the car's use has varied so much that the cost of maintaining the car varied considerably.	**20**	**CO5**

During the past 200 days, the demand for the car fluctuated as shown in the following table:

Trips per week	Frequency
0	16
1	24
2	30
3	60
4	40
5	30

Simulate the demand for a 10-week period by making use of the random numbers $82, 96, 18, 96, 20, 84, 56, 11, 52$ and 03. Also, find the Average demand per week.