Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES
End Semester Examination, May 2024

Course: Mathematical Methods
Program: B.Sc. Mathematics
Course Code: MATH 3033

Semester: VI
Time: 03 hrs.
Max. Marks: 100

Instructions:

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	**Solve** the following integral: $$I=\int_0^1 \sqrt{1-x^2}dx$$ using Trapezoidal and Simpson's 1/3 rules, when there are 10, 20 subintervals.	**2+2**	**CO4**
Q 2	**Define** the Dirichlet conditions. **Check** if the periodic function $$f(x)=\begin{cases}0, & -\pi < x \le 0 \\ \tan x, & 0 \le x < \pi\end{cases}$$ having period of 2π, satisfies the Dirichlet conditions, and if not, why?	**2+2**	**CO1**
Q 3	**Define** the *Regula Falsi* method and use it to **find** a root of $xe^x = \cos x$ upto three iteration.	**2+2**	**CO2**
Q 4	**Derive** the Gauss' Forward Interpolation Formula as well as the Stirling's Formula, starting from the Newton's interpolation formula $y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots + \frac{p(p-1)(p-2)\ldots(p-n+1)}{n!}\Delta^n y_0$. **Mention** how the indexing in the tabulated values is different for the Gauss' and Stirling's formula than for the Newton's.	**3+1**	**CO3**
Q 5	Use the trapezoidal rule for solving the integral $$I=\int_0^1\int_0^1 e^{x+y}dx\,dy$$ if the x and y values are equally spaced with the spacings being $h_x = h_y = 0.5$	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

Q 6	We can detect errors in tabular values using difference tables. Suppose there is an error of $+1$ unit in the sixth element of the tabular value that has 11 elements: $y_6^{(\text{tabulated})} = y_6^{(\text{true})} + 1$. Taking all values of y as 0, except y_6	**4+2+2+2**	**CO3**

<table>
<tr><td></td><td>which you can take as $+1$, write the difference table that has columns upto Δ^5 and answer the following questions:
1. How does the effect of the error change with increase in order of differences?
2. Can you connect the errors in any column with binomial coefficients?
3. What can you say about the algebraic sum of the errors in any column?</td><td></td><td></td></tr>
<tr><td>Q 7</td><td>Define the two-point, three-point and five-point formula for the first derivative of any function, whose tabulated values are provided, and derive the three-point formula.

Use the central difference formula to determine the second derivative for the data-set:

<table>
<tr><td>x</td><td>1</td><td>1.2</td><td>1.4</td><td>1.6</td><td>1.8</td><td>2.0</td><td>2.2</td></tr>
<tr><td>y</td><td>2.7183</td><td>3.3201</td><td>4.0552</td><td>4.9530</td><td>6.0496</td><td>7.3891</td><td>9.0250</td></tr>
</table>

at $x = 1.2$.</td><td>5+5</td><td>CO4</td></tr>
<tr><td>Q 8</td><td>Define the conditions for the existence of the Laplace transforms. Derive the initial value and final value theorems for the Laplace transforms. Derive the analytic expression of the Laplace transform for $f(t) = \sinh \omega t + \cosh \omega t$.</td><td>2+4+4</td><td>CO1</td></tr>
<tr><td>Q 9</td><td>Define the Newton Raphson's method and derive its order of convergence. Use the method to find a root for the equation $\sin x = \frac{x}{2}$, given that the root lies between $\frac{\pi}{2}$ and π upto the fourth iteration.

OR

Define the Secant method. Use this method to find a root for
i. $x^3 - 2x - 5 = 0$ with $x_0 = 2, x_1 = 3$ and three iterations
ii. $xe^x - 1 = 0$ with $x_0 = 0, x_1 = 1$ and three iterations</td><td>3+4+3

OR

2+4+4</td><td>CO2</td></tr>
<tr><td colspan="4">SECTION-C
(2Qx20M=40 Marks)</td></tr>
<tr><td>Q 10</td><td>a. Define numerical integration and discuss the Trapezoidal and Simpson's Rule, with associated errors.
b. Derive the numerical method of integration using the fourth order of differences – known as the Boole's rule.
c. Derive the associated error of the Boole's rule.</td><td>5+10+5</td><td>CO4</td></tr>
<tr><td>Q 11</td><td>a. Define the Lagrange Interpolation formula and prove that the Lagrange interpolating polynomial is unique.</td><td>5+5+5+5</td><td>CO3</td></tr>
</table>

	b. If we interchange the positions of x and y in the Lagrange interpolation formula, we obtain the inverse Lagrange interpolation formula. If $y_1 = 4, y_3 = 12, y_4 = 19, y_x = 7$, **find** x. c. **Mention** the error formula for a general interpolation of a function $f(x)$ with a polynomial $p(x)$. Use this to **find** the error bound for the Trapezoidal rule for numerical integration. d. If we are given values for $y = \ln x$ as follows: $y(x = 2) = 0.69315, y(x = 2.5) = 0.91629, y(x = 3) = 1.09861$, **find** the error bound on the value of y at $x = 2.7$ determined using the Lagrange Interpolation formula. **OR** a. **Define** the Bessel's and Everett's formula, highlighting the values of p for which these methods can be used. b. Beginning with the Bessel's formula and expressing the odd-order differences in terms of the just lower even-order differences, **derive** a relation between the Bessel's and Everett's formula. c. If we are given the following values for $y = e^x$: x: 0.61, 0.62, 0.63, 0.64, 0.65 y: 1.8404, 1.8589, 1.8776, 1.8965, 1.9155 use the Bessel's and Everett's formula to evaluate the function at $x = 0.634$.	**OR** **5+5+10**	