Name:

Enrolment No:

UPES
End Semester Examination, May 2024

Course: Mathematical Physics III | **Semester: IV**
Program: B.Sc. (Hons.) Physics | **Time: 03 hrs.**
Course Code: PHYS 2027 | **Max. Marks: 100**

Instructions:
1. All questions are compulsory (Q. No. 9 and Q. No. 11 have internal choices).
2. Scientific calculators can be used for calculations.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Define essential and removable singularities with examples in each case.	**4**	**CO1**
Q 2	Represent Dirac Delta function as a Fourier integral.	**4**	**CO2**
Q 3	State the shifting theorem for Laplace transform with an example.	**4**	**CO3**
Q 4	Prove that $\left\|e^{iz}\right\| = e^{-y}$ where symbols have their usual meaning.	**4**	**CO1**
Q 5	Define the orthogonality properties of Sine and Cosine functions in Fourier analysis.	**4**	**CO2**

SECTION B
(4Qx10M= 40 Marks)

Q 6	State the residue theorem. Find the poles and residue of the function $\frac{1}{z^m(1-z)^n}$ where m, n are positive integers. (3+7 = 10 marks)	**10**	**CO3**
Q 7	Suppose that f is a function of period 2 with $f(t) = t$ for $0 < t < 2$. Show that $f(t) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin n\pi t}{n}$ and sketch the graph of the function indicating the value at each discontinuity. Substitute an appropriate value of t to deduce the Leibniz's series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$ (7+3 = 10 marks)	**10**	**CO4**
Q 8	Prove that the Fourier transform of a Gaussian function is again a Gaussian function.	**10**	**CO2**
Q 9	Show that $\frac{n}{2\,cosh^2 nx}$ is Dirac-Delta function and independent of n.	**10**	**CO3**

	OR Use Laplace transform to solve: $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$, given that $x = 2, y = 0$ at $t = 0$.		
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	A resistance R is in series with inductance L and the combination is supported with emf $E(t)$. The current i is given by $L\frac{di}{dt} + Ri = E$. If the switch is connected at $t = 0$ and disconnected at $t = a$, find the current i as a function of t.	**20**	**CO4**
Q 11	Prove that $\int_0^\infty \sin x^2\, dx = \int_0^\infty \cos x^2\, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$. **OR** Evaluate $\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \frac{e^{zt}}{\sqrt{z+1}} dz$ where a and t are any positive constants.	**20**	**CO3**