Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2024

Course : Real Analysis II **Semester : IV**

Program : B. Sc. (H) Mathematics **Time : 03 hrs.**

Course Code: MATH2051 **Max. Marks: 100**

Instructions: Attempt all questions from Sections **A, B,** and **C**. Questions **6** and **10** have internal choices**.**

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	If f is a bounded function defined on $[a, b]$ and P be any partition of $[a, b]$, then show that $L(P, -f) = -U(P, f)$	**4**	**CO1**
Q 2	Compute $U(P, f)$ if $f(x) = x^2$ on $[0, 1]$ and $P = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$ be a partition of $[0, 1]$.	**4**	**CO1**
Q 3	Show that $\sum_{n=1}^{\infty} n^2 x^n$ is uniformly convergent in $[-\alpha, \alpha]$, when $0 < \alpha < 1$.	**4**	**CO2**
Q 4	Define uniform convergence of a sequence. State Cauchy's general principle of uniform convergence for series.	**4**	**CO2**
Q 5	State first and second form of Abel's theorem.	**4**	**CO3**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Prove that the function f defined on $[0, 4]$ by $f(x) = [x]$, where $[x]$ denotes the greatest integer not greater than x, is integrable on $[0, 4]$. Also find $\int_0^4 f(x)dx$. **OR** Find the upper and lower Riemann integrals for the function $f(x)$ defined on $[0, 1]$ as follows: $$f(x) = \begin{cases} (1 - x^2)^{1/2}, & when\ x\ is\ rational \\ 1 - x, & when\ x\ is\ irrational \end{cases},$$ and hence show that f is not R-integrable over $[0, 1]$.	**10**	**CO1**
Q 7	Show that the sequence $\langle f_n \rangle$, where $f_n(x) = nx(1 - x)^n$ is not uniformly convergent on $[0, 1]$.	**10**	**CO2**
Q 8	Show that the series $\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ converges uniformly.	**10**	**CO3**

Q 9	State and prove Taylor's theorem for power series.	**10**	**CO3**
	SECTION-C **(2Qx20M=40 Marks)**		
Q 10	Prove that $\sum \frac{x}{n^p+n^q x^2}$, $p > 1$ is uniformly convergent for all real x if $p + q > 2$. **OR** Test for uniform convergence and term by term integration of the series $\sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2}$. Also prove that $\int_0^1 \left(\sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2}\right) dx = \frac{1}{2}$.	**10+10**	**CO2**
Q 11	**(i)** Find the radius of convergence and exact interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(n+1)}{(n+1)(n+3)} x^n$. **(ii)** Show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots,\ -1 < x \leq 1$ and hence deduce that $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$.	**20**	**CO3**