Name: **Enrolment No:**	

UPES

End Semester Examination, December 2023

Course: Numerical Methods in Scientific Computing **Semester: VII**

Program: BSc (Physics) by Research **Time : 03 hrs.**

Course Code: PHYS 4024P **Max. Marks: 100**

Instructions:

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Discuss the importance of interpolation in Scientific Computing. What is the error in polynomial interpolation schemes?	**4**	**CO1**
Q2	Using finite differencing, write the expressions for following derivatives: a) $\frac{\partial^2 f}{\partial x^2}$ b) $\frac{\partial^2 f}{\partial x \partial y}$	**4**	**CO1**
Q3	What is a positive definite matrix? Check if the given matrix is positive definite or not? $A = \begin{bmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}$	**4**	**CO1**
Q4	Differentiate between composite integration and Gauss quadrature.	**4**	**CO1**
Q5	Find the spectral radius of the following matrix: $\begin{bmatrix} 3 & 2 & -1 \\ 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix}$	**4**	**CO1**

SECTION B

(4Qx10M= 40 Marks)

Q6	Using Hermite interpolation, approximate a function passing through the following data: x | $f(x)$ | $f'(x)$ 0.1 | −0.29004996 | −2.8019975 0.2 | −0.56079734 | −2.6159201 0.3 | −0.81401972 | −2.9734038 If $P(x)$ is the approximate polynomial, find $P(0.13)$. **OR**	**10**	**CO3**

	Construct a Natural cubic spline for the following data: x | $f(x)$ 0.1 | −0.29004996 0.2 | −0.56079734 0.3 | −0.81401972 If $Q(x)$ is the approximate function passing through the above data, find $Q(0.25)$		
Q7	A particle of mass m moving through a fluid is subjected to a viscous resistance R, which is a function of the velocity v. The relationship between the resistance R, velocity v, and time t is given by the equation $$t = \int_{v(t_0)}^{v(t)} \frac{m}{R(u)} du$$ Suppose that $R(u) = -v\sqrt{v}$ for a particular fluid, where R is in newtons and v is in m/s. If $m = 10$ kg and $v(t_0) = 10$ m/s, approximate the time required for the particle to slow to $v = 5$ m/s.	**10**	**CO4**
Q8	Water flows from an inverted tank with circular orifice at the rate $$\frac{dx}{dt} = -0.6\,\pi r^2 \sqrt{2g}\,\frac{\sqrt{x}}{A(x)}$$ where r is the radius of the orifice, x is the height of the liquid from the vertex of the cone, and $A(x)$ is the area of the cross-section of the tank with x units above the orifice. Suppose $r = 0.1$ ft, $g = 32.1$ ft/s^2, and the tank has an initial water level of 8 ft and initial volume $512(\pi/3)$ ft^3. Use RK2 method to find the water level after 10 minutes with $h =$ 20 sec.	**10**	**CO4**
Q9	Use Newton method to solve the following system of non-linear equations: $$6x_1 - 2\cos(x_2 x_3) - 1 = 0$$ $$9x_2 + \sqrt{x_1^2 + \sin x_3 + 1.06} + 0.9 = 0$$ $$60x_3 + 3e^{-x_1 x_2} + 10\pi - 3 = 0$$ Find out approximations to the solution until $\left\lVert \boldsymbol{x}^{(k)} - \boldsymbol{x}^{(k-1)} \right\rVert_\infty \leq 10^{-4}$. Use the guess vector as $\boldsymbol{x}^{(0)} = (0,0,0)^t$.	**10**	**CO2**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	(**a**) Using 4-step Adam's Bashforth method, solve the following time-dependent ODE: $$\frac{dy}{dt} = 1 + \frac{y}{t}, \quad 1 \leq t \leq 2, \quad y(1) = 2$$ Take $h = 0.2$. The actual solution is $y(t) = t\ln t + 2t$ (**10 Marks**) (**b**) Discretize the Poisson Equation given below using finite difference method:		**CO4**

	$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=4 \qquad 0<x<1, \quad 0<y<1$$ $$u(x,0)=x^2,\ \ u(x,1)=(x-2)^2, \qquad 0\le x\le 1$$ $$u(0,y)=y^2, \qquad u(1,y)=(y-1)^2,\ \ 0\le y\le 1$$ Use $h=k=\frac{1}{4}$. Write the discretized equations in terms of $A\boldsymbol{u}=\boldsymbol{b}$, where A is the coefficient matrix, $\boldsymbol{u}$ is the unknown and $\boldsymbol{b}$ is known vector. Discuss the solution strategy. (**10 Marks**) **OR** (**a**) Using 3-step Adam's Moulton method to solve the following ODE: $$\frac{dy}{dt}=\cos 2t+\sin 3t\,, \quad 0\le t\le 1, \quad y(0)=1$$ Take $h=0.2$. The actual solution is $y(t)=\frac{1}{2}\sin 2t-\frac{1}{3}\cos 3t+\frac{4}{3}$ (**10 Marks**) (**b**) Discretize the Laplace equation given below using Finite difference method: $$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0 \qquad 1<x<2, \quad 0<y<1$$ $$u(x,0)=2\ln x\,,\ \ u(x,1)=\ln(x^2+1)\,,\ \ 1\le x\le 2$$ $$u(1,y)=\ln(y^2+1)\,,\ \ u(2,y)=\ln(y^2+4)\,,\ \ 0\le y\le 1$$ Use $h=k=\frac{1}{3}$. Write the discretized equations in terms of $A\boldsymbol{u}=\boldsymbol{b}$, where A is the coefficient matrix, $\boldsymbol{u}$ is the unknown and $\boldsymbol{b}$ is known vector. Discuss the solution strategy. (**10 Marks**)		
Q11	The temperature $u(x,t)$ of a long, thin rod of constant cross section and homogeneous conducting material is governed by 1-D heat equation. If heat is generated in the material, for example, by resistance to current or nuclear reaction, the heat equation becomes: $$\frac{\partial^2 u}{\partial x^2}+\frac{Kr}{\rho C}=K\frac{\partial u}{\partial t}, \quad 0<x<l,\ \ 0<t$$ where l is the length, ρ is the density, C is the specific heat, and K is the thermal diffusivity of the rod. The function $r=r(x,t,u)$ represents the heat generated per unit volume. Suppose that $l=1.5$ cm, $K=1.04$ cal/cm-deg-sec, $\rho=10.6$ g/cm^3, $C=0.056$ cal/g-deg and $r(x,t,u)=5.0$ cal/cm^3-s If the ends of the rod are kept at 0^{o}C, then $$u(0,t)=u(l,t)=0, \quad t>0$$	**20**	**CO4**

	Suppose the initial temperature distribution is given by $$u(x,0) = \sin\frac{\pi x}{l}, \quad 0 \le x \le l$$ Using the above information, approximate the temperature distribution with $h = 0.15$ and $k = 0.0225$		