Name: **Enrolment No:**	UPES University of Tomorrow

UPES

End Semester Examination, December 2023

Program Name: BSc (Physics Honors) — **Semester : V**
Course Name: Monte Carlo Methods — **Time : 3 hrs**
Course Code: PHYS 3029 — **Max. Marks: 100**
Nos. of page(s): 2

Instructions: Attempt all questions

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Discuss linear congruential method to generate random numbers? Why are such numbers called pseudo random numbers?	4	CO1
Q 2	Describe Box-Muller approach to generate the normal random variables	4	CO2
Q 3	Define a Markov process. What are the key properties that distinguish it from other stochastic processes?	4	CO1
Q 4	Suppose that X and Y are independent binomial random variables with parameters (n, p) and (m, p). i. Find $E[e^{X+Y}]$ ii. Outline the MC procedure to $E[e^{X+Y}]$	4	CO2
Q 5	Outline the procedure to generate exponentially distributed random numbers with parameter λ.	4	CO3

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	a. Provide an example of birth and death process in biological modeling. b. Describe the Chapman – Kolmogorov equation and explain how it can be used to study the radioactivity? c. Discuss random walk problem. How will you simulate sample paths of the random walk. Outline the salient features of the algorithm.	10	CO1
Q7	a. Define a stochastic process. How do you classify it ? b. Define stationary and wide sense stationary process. Show that for a stationary stochastic process X(t), $t \in T$, $E[X(t)]$ is constant.	10	CO2

	c. Define a Brownian motion process. What can you say about its correlation structure . d. Consider a stochastic process X(t), t ∈ T defined by X(t) = a cos(ωt+ Φ) where a and ω are constants and is a uniformly distributed random variable in the interval [0, 2 π] i. Simulate the sample paths ii. How will you estimate the E[X(t)] iii. Is X(t) a wide-sense stationary stochastic process ?		
Q 8	Write pseudocode for the Metropolis-Hastings algorithm to sample from a given probability distribution. Include details on initializing the algorithm and determining convergence.	**10**	**CO2**
Q 9	a. Define an absorbing Markov chain. Provide a method to determine the absorbing probabilities and expected number of steps before absorption for a given state. b. Explain the concept of transition probabilities in a Markov chain. How do they determine the behavior of the chain?	**10**	**CO1**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	a. Show that the sum of independent identically distributed exponential random variables has a gamma distribution. b. Calculate the moment generating function of the uniform distribution on (0, 1). Obtain E[X] and Var[X] by differentiating. c. State the central limit theorem. How will you illustrate this theorem using uniformly distributed random numbers.	**20**	**CO2**
Q 11	a. Define a Poisson process. What are the basic assumptions? Give two real life examples of the Poisson Process . b. Three friends A, B and C decide to meet at a certain place between 10:00 AM to 11:00 AM. Each will arrive randomly and wait for only 5 minutes. Using Monte Carlo method, outline the procedure to estimate the probability of meeting. c. Given a simple Markov chain with 3 states, construct the state transition matrix and interpret its meaning.	**20**	**CO3**