Name:

Enrolment No:

UPES

End Semester Examination, December 2023

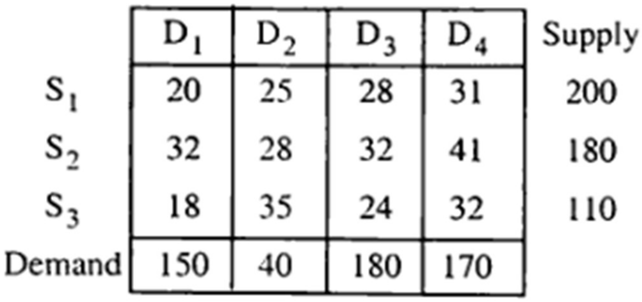
Course: Linear and Nonlinear Programming **Semester: V**
Program: B.Sc. (Hons.) Mathematics **Time : 03 hrs.**
Course Code: MATH3032 **Max. Marks: 100**
Instructions: Answer all the questions.

SECTION A
(5Qx4M=20Marks)

<table>
<tr><td>S. No.</td><td></td><td>Marks</td><td>CO</td></tr>
<tr><td>Q 1</td><td>Define (a) Solution (b) Feasible solution (c) Basic solution and (d) Unbounded solution of a LPP.</td><td>4</td><td>CO1</td></tr>
<tr><td>Q 2</td><td>Write the dual of the
$Min\ Z = 2y + 5z$ subject to
$$x + y \geq 2$$
$$2x + y + 6z \leq 6$$
$$x - y + 3z = 4$$
and $x, y, z \geq 0$</td><td>4</td><td>CO1</td></tr>
<tr><td>Q 3</td><td>Discuss the mathematical formulation of the transportation problem.</td><td>4</td><td>CO2</td></tr>
<tr><td>Q 4</td><td>Using Least Cost Method, find the initial basic feasible solution to the following transportation problem.
<table>
<tr><td rowspan="2">Factory</td><td></td><td></td><td></td><td></td><td rowspan="2">Supply</td></tr>
<tr><td>D_1</td><td>D_2</td><td>D_3</td><td>D_4</td></tr>
<tr><td>S_1</td><td>19</td><td>30</td><td>50</td><td>10</td><td>7</td></tr>
<tr><td>S_2</td><td>70</td><td>30</td><td>40</td><td>60</td><td>9</td></tr>
<tr><td>S_3</td><td>40</td><td>8</td><td>70</td><td>20</td><td>18</td></tr>
<tr><td>Demand</td><td>5</td><td>8</td><td>7</td><td>14</td><td></td></tr>
</table>
</td><td>4</td><td>CO2</td></tr>
</table>

<table>
<tr><td>Q 5</td><td>The efficiency E of a small manufacturing concern depends on the workers W and is given by $10E = -\left(\frac{w^3}{40}\right) + 30W - 392$. Find the strength of the workers that would give the maximum efficiency.</td><td>4</td><td>CO3</td></tr>
<tr><td colspan="4">SECTION B
(4Qx10M= 40 Marks)</td></tr>
<tr><td>Q 6</td><td>Solve the following LPP using Dual Simplex method.

$Min\ Z = 3x + y$ subject to
$x + y \geq 1$
$2x + 3y \geq 2$
and $x, y \geq 0$.</td><td>10</td><td>CO1</td></tr>
<tr><td>Q 7</td><td>Solve the following transportation problem by VAM method and find the minimum cost.


<table>
<tr><td></td><td>D₁</td><td>D₂</td><td>D₃</td><td>D₄</td><td>Supply</td></tr>
<tr><td>S₁</td><td>20</td><td>25</td><td>28</td><td>31</td><td>200</td></tr>
<tr><td>S₂</td><td>32</td><td>28</td><td>32</td><td>41</td><td>180</td></tr>
<tr><td>S₃</td><td>18</td><td>35</td><td>24</td><td>32</td><td>110</td></tr>
<tr><td>Demand</td><td>150</td><td>40</td><td>180</td><td>170</td><td></td></tr>
</table></td><td>10</td><td>CO2</td></tr>
<tr><td>Q 8</td><td>Discuss the Kuhn-Tucker necessary conditions and obtain the Kuhn-Tucker necessary conditions for the following problem.
$Max\ Z = 10x - x^2 + 10y - y^2$ subject to
$x + y \leq 9$
$x - y \geq 6$
and $x, y \geq 0$.</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 9</td><td>Find the second order Taylor's series expansion of the function $f(x, y) = 12xy + 5y^2$ about the point $[1,0]^T$.

(OR)

Consider the function $f(x, y) = 3x^2 + y^2 - 10$. Determine the maximum or minimum point (if any) of the function.</td><td>10</td><td>CO3</td></tr>
</table>

SECTION-C
(2Qx20M=40 Marks)

Q 10

Use penalty (Big-M) method to solve the following LPP.

$Minimize\ Z = 5x + 3y$ subject to

$$2x + 4y \leq 12$$
$$2x + 2y = 10$$
$$5x + 2y >= 10$$

and $x, y \geq 0$

(OR)

Solve the following Travelling salesman problem.

From/To	A	B	C	D	E
A	–	3	6	2	3
B	3	–	5	2	3
C	6	5	–	6	4
D	2	2	6	–	6
E	3	3	4	6	–

20

CO2

Q 11

Use the method of Lagrangian multipliers to solve the following NLP problem. Does this solution maximize or minimize the objective function?

$Optimize\ Z = 4x^2 + 2y^2 + z^2 - 4xy$ subject to

$$x + y + z = 15$$
$$2x - y + 2z = 20$$

and $x, y, z \geq 0$

20

CO3