Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2023

Course: Analytical Geometry **Semester: III**
Program: B.Sc. (H) Mathematics **Time : 03 hrs.**
Course Code: MATH 2047 **Max. Marks: 100**

Instructions: Read all the below mentioned instructions carefully and follow them strictly:

1) Mention Roll No. at the top of the question paper.
2) Attempt all the parts of a question at one place only.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Prove that the centres of the circles $x^2+y^2+4y+3=0$, $x^2+y^2+6x+8y-17=0$ and $x^2+y^2-30x-16y-42=0$ are collinear.	**4**	**CO1**
Q 2	Find the equation of the tangent and normal at t on the parabola $x^2=4ay$.	**4**	**CO2**
Q 3	Show that the point (8, 9) lies on the circle $x^2+y^2-10x-12y+43=0$ and find the other end of the diameter through (8, 9).	**4**	**CO3**
Q 4	Derive the equation of tangent at (α,β) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.	**4**	**CO3**
Q 5	Find the equation of the hyperbola whose focus is (2, 2), eccentricity $\frac{3}{2}$ and directrix $3x-4y=1$.	**4**	**CO2**
	SECTION B **(4Qx10M= 40 Marks)**		
Q 6	Find the equations of the right circular cylinder of radius 3 with equations of axis $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$.	**10**	**CO4**
Q 7	Obtain the equation to the tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ which are parallel to the plane $lx+my+nz=0$.	**10**	**CO3**
Q 8	Show that the plane $2x-2y+z+12=0$ touches the conic $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.	**10**	**CO2**
Q 9	The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the coordinate axes in A, B and C. Find the equation of the sphere passing through A, B, C and O. Also find its center and radius. **OR**	**10**	**CO1**

	The equations to AB are $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ through a point P (1, 0, -1), PN is drawn perpendicular to AB, and PQ is drawn parallel to the plane $3x + 4y + 5z = 0$ to meet AB in Q. Find the equations of PN and PQ.		
SECTION-C (2Qx20M=40 Marks)			
Q 10	Find the angle between the lines of section of the plane $3x + y + 5z = 0$ and the cone $6yz - 2zx + 5xy = 0$.	**20**	**CO4**
Q 11	The rods whose lengths are a and b slide along the coordinate axes in such a way that their extremities are concyclic. Find the locus of the center of the circle. **OR** A common tangent is drawn to the circle $x^2 + y^2 = r^2$ and the parabola $y^2 = 4ax$. Show that the angle θ which it makes with the axis of the parabola is given by $tan^2\theta = \frac{\sqrt{r^2+4a^2}-r}{2r}$.	**20**	**CO4**