Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2023

Course: Linear Algebra | **Semester: I**
Program: B.Sc. (H) Mathematics | **Time : 03 hrs.**
Course Code: MATH1057 | **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Find the rank of the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 2 \end{bmatrix}$.	**4**	**CO1**
Q 2	Define Internal and external composition of vector space.	**4**	**CO2**
Q 3	Show that the vectors $(1,2,-2), (-1,3,0), (0,-2,1)$ are linearly independent vectors.	**4**	**CO2**
Q 4	Prove that the intersection of two subspaces of a vector space is also a subspace.	**4**	**CO2**
Q 5	Let F be the field of the complex numbers and let T be the function from R^3 to R^3 defined by $T(a_1, a_2, a_3) = (a_1 - a_2 + 2a_3, 2a_1 + a_2 - a_3, -a_1 - 2a_2)$ then show that T is a linear transformation.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Show that the set of numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers, is a field with respect to addition and multiplication.	**10**	**CO2**
Q 7	Define the linear sum of two subspaces. Prove that if W_1 and W_2 are subspaces of a vector space $V(F)$ then $W_1 + W_2$ is also a subspace of $V(F)$.	**10**	**CO2**
Q 8	Let U and V be two finite dimensional vector spaces over the same field F and let $\{\alpha_1, \alpha_2, \dots \dots \dots . \alpha_n\}$ be ordered basis for U and let $\{\beta_1, \beta_2, \dots \dots \dots . \beta_n\}$ be ordered basis for V then prove that there is precisely one linear transformation $T: U \to V$ such that $T(\alpha_j) = \beta_j, j = 1,2,3 \dots \dots n$.	**10**	**CO3**

Q 9	Show that the homogenous system of equations: $x + y\cos\gamma + z\cos\beta = 0,$ $x\cos\gamma + y + z\cos\alpha = 0,$ $x\cos\beta + y\cos\alpha + z = 0$ has non-trivial solution if $\alpha + \beta + \gamma = 0$. **OR** Find values of λ for which the following system of equations is consistent and non-trivial solutions. Solve equations for all such values of λ. $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$ $(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$ $2x + (3\lambda + 1)y - 3(\lambda - 1)z = 0$	**10**	**CO1**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Find the modal matrix P such that $P^{-1}AP$ is diagonal matrix, where $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. **OR** State Cayley Hamilton theorem. Verify it for matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$. Hence find A^{-1}.	**20**	**CO1**
Q 11	Let U and V be the vector spaces over the same field F and let T be a linear transformation from U to V where U is finite dimensional then prove that $rank(T) + nullity\ (T) = \dim(U)$.	**20**	**CO3**