Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2023

Course: Metric Spaces & Complex Analysis **Semester: VI**
Program: B.Sc. (H) Mathematics **Time : 03 hrs.**
Course Code: MATH 3005 **Max. Marks: 100**

Instructions: Attempt all the questions. Each question in Section A carries 4 marks. Each question in Section B carries 10 marks. Each question in Section C carries 20 marks. Internal choices are available for questions 9 and 11.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Give an example of a pseudo-metric space which is not a metric space.	**4**	**CO1**
Q 2	Evaluate $\oint_C \frac{1}{z}dz$, where C is the circle defined by $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$.	**4**	**CO4**
Q 3	Find the circle and radius of convergence of the given power series: $$\sum_{k=0}^{\infty} \frac{1}{(1-2i)^{k+1}}(z-2i)^k$$	**4**	**CO3**
Q 4	Discuss the nature of singularity of $f(z) = \frac{z-\sin z}{z^3}$ at $z = 0$.	**4**	**CO3**
Q 5	Find the residue at $z = 0$ of the following function: $$f(z) = \frac{1+e^z}{\sin z + z\cos z}$$	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Using Cauchy's integral formula, evaluate $\oint_C \frac{z+1}{z^4+2iz^3}dz$, where C is the circle $\lvert z\rvert = 1$.	**10**	**CO4**
Q 7	Describe the open sphere of unit radius about $(0, 0)$ of the following metric for $\mathbf{R^2}$: $$d(z_1, z_2) = \lvert x_1 - x_2\rvert + \lvert y_1 - y_2\rvert,$$ where $z_1 = (x_1, y_1)$, $z_2 = (x_2, y_2)$ are any two points of $\mathbf{R^2}$.	**10**	**CO1**
Q 8	Using the method of contour integral, evaluate the given real trigonometric integral $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}d\theta$.	**10**	**CO4**

Q 9	Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for $1 < \|z-2\| < 2$. **OR** Find the maximum modulus of $f(z) = 2z + 5i$ on a closed circular region defined by $\|z\| \leq 2$.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	(a) Prove that the function $f: [0, b] \to \boldsymbol{R}$ defined by $f(x) = x^2$ is uniformly continuous, where $b > 0$. (b) Let C be the set of all real valued bounded continuous function defined on [0, 1]. If d is defined by $d(f, g) = \sup\{\|f(x) - g(x)\|: x \in [0, 1]\}$, then show that d is a metric on C, where $f, g \in C$.	**10+10**	**CO2**
Q 11	State and prove Taylor's theorem for the complex function $f(z)$. **OR** State and prove Cauchy's residue theorem. Using the theorem, evaluate $\oint_C \tan z \; dz$, where C is the circle $\|z\| = 2$.	**20**	**CO5**