Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2023

Course: Function of several variables and Partial differential equations **Semester: IV**

Program: B. Sc.(H)/Int. B.Sc-M.Sc. Mathematics **Time : 03 hrs.**

Course Code: MATH 2050 **Max. Marks: 100**

Instructions:

1. Section A has 5 questions. All questions are compulsory.
2. Section B has 4 questions. All questions are compulsory. Question 8 has internal choice to attempt anyone.
3. Section C has 2 questions. All questions are compulsory. Question 11 has internal choice to attempt anyone.

SECTION A

(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Define partial derivatives for a function of two variables. Give an example of a function which is not continuous, but all its partial derivatives exist.	**4**	**CO1**
Q 2	Solve the PDE: $(D^3 - 3D^2D' + 4D'^3)u = 0.$	**4**	**CO2**
Q 3	Determine the region in which the given equation is hyperbolic, parabolic, or elliptic. $$U_{xx} + y^2U_{yy} = y.$$	**4**	**CO3**
Q 4	Determine if the given PDE is reducible or irreducible with justification. $$(D^2 - D'^2 + D - D')u = 0$$	**4**	**CO3**
Q 5	Find general integral of the PDE. $$\frac{y^2z}{x}p + xzq = y^2.$$	**4**	**CO2**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Find local maxima, local minima and saddle point of the function. $$f(x, y) = 3x^2 + 6xy + 7y^2 - 2x + 4y$$	**10**	**CO1**
Q 7	Solve the following PDE $$(D^2 - 4DD' + 4D'^2)u = e^{2x+y}$$	**10**	**CO2**

Q 8	Reduce the equation to canonical form $$(\mathrm{n}-1)^2 u_{xx} - y^{2n} u_{yy} = n y^{2n-1} u_y$$ **OR** Find complete integral of the PDE: $$(D^2 - DD' - 2D)u = \sin(3x + 4y)$$	**10**	**CO3**
Q 9	Obtain the solution of the wave equation $u_{tt} = 5\, u_{xx}$ under the following conditions: (i) $u(0,t) = u(2,t) = 0$ (ii) $u(x,0) = sin(\frac{3\pi x}{2})$ (iii) $u_t(x,0) = 0$	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Obtain the complete integral of the given PDE $(D^2 - DD' + D' - 1)u = \cos(x + 2y) + e^{x+y} + xy$	**20**	**CO3**
Q11	Discuss all possible solutions of Laplacian equations using variable separable method. $$U_{xx} + U_{yy} = 0$$ **OR** A bar of 100cm long, with insulated sides, has its ends kept at $0^o C$ and $100^o C$ until steady state conditions prevail. The two ends are then suddenly insulated and kept so. Find the temperature distribution.	**20**	**CO4**