Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End-Semester Examination, May 2023

Course : Mathematical Physics -III **Semester : IV**
Program : B. Sc. (Hon.) **Time : 03 hrs.**
Course Code: (PHYS 2027) **Max. Marks: 100**

Instructions:

- All questions are compulsory (Q. 9 and Q. 11 have internal choice)
- Scientific calculators can be used for calculations

SECTION A

(5Qx4M=20Marks)

S. No.	Attempt all Questions (Short answer type)	**Marks**	**CO**
Q.1	Find the roots of the complex Equation: $Z^5 = 5$	**04**	**CO1**
Q.2	State the "Convolution Theorem" in Fourier Transform. Find the convolution of the functions f(x) and g(x) given by: $f(x) = \delta(x-a)$ and $g(x) = \sin(x)$: a is a constant and δ is the Dirac delta.	**04**	**CO2**
Q.3	Given that a periodic function f(x) is expanded in Fourier Series $f(x) = a_0/2 + \sum_1^{\infty} a_n \cos(nx) + \sum_1^{\infty} b_n \sin(nx)$ where, a_0, a_n and b_n have usual meaning. If $C_n = (a_n - i.b_n)/2$, prove that i) $C_{-n} = (a_n + ib_n)/2$ and ii) $C_o = a_o/2$	**04**	**CO1**
Q.4	Prove that the Laplace transform of a periodic function f(t) with periodicity T, is $\{F_o(S)/(1-\exp(-TS)\}$: Where $F_o(S) = \int_0^T f(t)e^{-St}\, dt$	**04**	**CO3**

Q.5	Given L(ω) is the Laplace Transform for a function f (x). Write/find the expression for the Laplace Transform of the function f (a.x); where 'a' is a constant.	**04**	**CO2**
	SECTION B **(4Qx10M= 40 Marks)**		
	Attempt all questions. Please note that Q.9 has a choice.		
Q.6	Find the Fourier Transform of the function! $f(t) = e^{ibt}$: $-a < t < a$ $= 0$: otherwise	**10**	**CO3**
Q.7	a) Comment on the singularity of the function $f(Z) = \{3Z^3/(Z^2 + 3^2)^2\}$ b) Evaluate the integral $$\oint_c f(Z).dz$$ around 'C' given by the closed path $\|Z - 2i\| = 3$. f(Z) is given in part a), above.	**10**	**CO1**
Q.8	Given that the Laplace transform of 1 {that is $L(1)$} = 1/S. Staring from this find the Laplace transform of a) t^n : [Hint: use the property frequency differentiation] b) e^{at} : [Hint: use the property frequency shift]	**10**	**CO3**
Q.9	**Attempt any one (Either I or II)** I. A Fourier Series for a function f(x) is given as $f(x) = a_0/2 + \sum_1^{\infty} a_n \cos(nx) + \sum_1^{\infty} b_n \sin(nx)$ What should be the condition/conditions imposed on the above series so that we can perform term by term a) Integration and b) differentiation **OR** II. Find Fourier Transform (U (k, t)) of the function u(x, t), which satisfies the Partial Differential Equation: $u_{xx} = u_t$; where $u_{xx} = \partial^2 u(x,t) / \partial x^2$ and $u_t = \partial u / \partial t$	**10**	**CO2** **CO4**

	Given u (x,0) = δ (x), where δ (x) is the Dirac delta function.		
SECTION-C **(2Qx20M=40 Marks)**			
	Attempt all questions. Please note that Q. 11 has a choice.		
Q.10	Use Laplace Transform to solve the following Ordinary Differential Equation: $y'' - 3y' + 2y = \exp(3t)$; where $y' = dy(t)/dt$. Etc. Initial Conditions: $y(0) = 1$ and $y'(0) = 0$.	**20**	**CO4**
Q.11	**Attempt any one (Either I or II):** I. Find the Fourier series for a Saw-Tooth function f(x) given by $f(x) = 2\pi - x$ for $0 < x < 2\pi$ And $f(x) = f(x+2\pi)$ **OR** II. Evaluate the Fourier transform of the following functions: a) $\exp(-ax^2)$: $a > 0$ b) $\sin(ax)$: $a > 0$	**20**	**CO3** **CO3**