Name:

Enrolment No:

UPES

End Semester Examination, May 2023

Course: Probability and Statistics **Semester: IV**

Program: B.Sc (H) Mathematics / Int. B.Sc M.Sc Mathematics **Time: 03 hrs.**

Course Code: MATH2052 **Max. Marks: 100**

Instructions: Attempt All Questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Let $\Omega = 1,2,3,4$ be a sample space. Check whether the set $\mathcal{F} = \{\phi, \Omega, \{1\}, \{1,2\}, \{3,4\}\}$ is a sigma field. If your answer is no, then find the smallest sigma field containing $\mathcal{F}$.	**4**	**CO1**
Q 2	Two dice are thrown together. What is the probability that the number obtained on one of the dice is multiple of number obtained on the other dice?	**4**	**CO1**
Q 3	Let X be a discrete random variable taking values in $\{-3,-2,-1,0,1,2,3\}$ such that $P[X=-3] = P[X=-2] = P[X=-1] = P[X=1] = P[X=2] = P[X=3]$ and $P[X<0] = P[X=0] = P[X>0]$. Find the cumulative distribution function of X.	**4**	**CO1**
Q 4	In a family of 5 children, what is the probability that there will be more boys than girls? (Use Binomial and assume that the probability of having a boy or a girl is ½ each).	**4**	**CO2**
Q 5	If X is a random variable with $E[X] = 3$ and $E[X^2] = 13$ then determine a lower bound for $P(-2 < X < 8)$ using Chebyshev's inequality.	**4**	**CO3**
	SECTION B **(4Qx10M= 40 Marks)**		
Q 6	Let X be a random variable that gives number of die tosses required to get the first 6. Then write the range of X, p.m.f. (probability mass function), mean, variance, and evaluate $P(X \geq 1.5)$, $P(0 < X < 4)$, and $P(X = 2 \mid X < 5)$.	**10**	**CO1**
Q 7	The following function defines a p.d.f. (probability density function) for some $\alpha > 0$? $$f(x) = \begin{cases} \frac{1}{\alpha x^2}, & if\ |x| > 1 \\ 0, & otherwise \end{cases}$$ Calculate the value of α, the corresponding CDF, and MGF. Also evaluate the expectation and variance of the corresponding random variable.	**10**	**CO1**
Q 8	Consider two random variables X and Y with joint PMF given in the following table.	**10**	**CO2**

		$Y=2$	$Y=4$	$Y=5$	
	$X=1$	$1/12$	$1/24$	$1/24$	
	$X=2$	$1/6$	$1/12$	$1/8$	
	$X=3$	$1/4$	$1/8$	$1/12$	
	Calculate $P(X \leq 2, Y \leq 4)$, formulate the marginal p.m.f. of X and Y, and evaluate $P(Y=2\|X=1)$. Are X and Y independent?				
Q 9	The length of life of an instrument produced by a machine has a normal distribution with a mean of 12 months and standard deviation of 2 months. Calculate the probability that an instrument produced by this machine will last (a) less than 7 months. (b) between 7 and 12 months. **OR** In a normally distributed data, 31% of the items are under 45, and 8% of the items are over 64. Find the mean and variance of the distribution. Also estimate the percentage of data (a) between 55 and 65 (b) between 45 and 60.			**10**	**CO3**
	SECTION-C **(2Qx20M=40 Marks)**				
Q 10	Random variables X and Y have joint continuous distribution with p.d.f. $f_{XY}(x,y) = \begin{cases} c(2x+y), & if\ 0<x<1, 0<y<2 \\ 0, & otherwise \end{cases}$ (a) Find the value of c. (b) Compute $E[X]$ and $E[Y]$. (c) Formulate $E[X\|Y=y]$.			**20**	**CO2**
Q 11	The data on heights of fathers and sons are given in the following table, where X is height of fathers and Y is height of son. X: 65, 63, 67, 64, 68, 62, 70, 66, 68, 67 Y: 68, 66, 68, 65, 69, 66, 68, 65, 71, 67 Evaluate the coefficient of correlation for the following data by making a table. Also construct a regression line and consequently predict the son's height for father with height 72 inches. **OR** Suppose that the table given below represents the data on two random variables X and Y. X: 1, 2, 3, 4, 5, 6, 7 Y: 9, 8, 10, 12, 11, 13, 14 Calculate the regression coefficient and obtain the lines of regression for the following data. Then predict the Y value for $X=4.5$. Also compute the correlation coefficient of X and Y.			**20**	**CO3**