


Name: Enrolment No:	
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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2023

Program Name : B. Sc. (H) Mathematics	Semester : II
Course Name : Integral Calculus	Time : 03 Hrs.
Course Code : MATH-1046	Max Marks : 100
Nos. of page(s) : 02	

Instructions:

Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 8 and 11 have internal choice.

SECTION A

S. No.		Marks	CO
Q 1	Evaluate the integral $\int_0^{\pi/4} \sin^4 2x dx$.	4	CO1
Q 2	Show that $(I_{n-1} + I_{n+1}) = 1$, where $I_n = \int_0^{\pi/4} \tan^n \theta d\theta$.	4	CO2
Q 3	Find the value of $\Gamma\left(-\frac{5}{2}\right)$.	4	CO3
Q 4	If $u = x \sin y$ and $v = y \sin x$, then find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$.	4	CO4
Q 5	Find the volume generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a, b > 0)$ about the x -axis.	4	CO5

SECTION B

Q 6	Evaluate: $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \frac{3}{n^2} \sec^2 \frac{9}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right]$	10	CO1
Q 7	Derive a reduction formula for $\int e^{ax} \sin^n x dx$ and hence evaluate $\int e^x \sin^3 x dx$.	10	CO2
Q 8	Change the order of integration and hence evaluate $\int_0^1 \int_x^1 (x^2 + y^2) dy dx$.	10	CO4
	OR		
	Evaluate $\int \int \frac{2xy^5}{\sqrt{1+x^2y^2-y^4}} dx dy$ over the triangle having vertices (0, 0), (1, 1) and (0, 1).		
Q 9	Find the length of the arc of the curve $y = \log_e \left(\frac{e^x - 1}{e^x + 1} \right)$ from $x = 1$ to $x = 2$.	10	CO5

SECTION-C

Q 10	(i) Prove that: $\beta(m, n) = \beta(m + 1, n) + \beta(m, n + 1)$ (ii) Express $\int_0^1 x^m(1 - x^p)^n dx$ in terms of Beta function and hence evaluate the integral $\int_0^1 x^{3/2}(1 - \sqrt{x})^{1/2} dx$.	10+10	CO3
Q 11	Find the area of the curvilinear triangle bounded by the parabolas $y^2 = 12x$, $x^2 = 12y$, circle $x^2 + y^2 = 45$ and lying outside the circle. OR Using the transformation $x - y = u$, $x + y = v$, evaluate $\iint \cos\left(\frac{x-y}{x+y}\right) dx dy$ over the region bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$.	20	CO5