Name: **Enrolment No:** **SAP ID:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2023

Program: B.Sc. (H.) Mathematics | **Semester: II**
Course: Real Analysis I | **Time: 3 Hours**
Course Code: MATH 1045 | **Max. Marks: 100**

Instructions: Answer All the questions.

SECTION A **(5x4 = 20 marks)**

Q 1	Show that every finite set is a closed set.	**4**	**CO1**
Q 2	Show that $<\frac{1}{n}>$ is a convergent sequence.	**4**	**CO2**
Q 3	Show that if $\{x_n\}$ converges to l, then $\{\lvert x_n\rvert\}$ converges to $\lvert l\rvert$. What about the converse?	**4**	**CO2**
Q 4	Determine the convergence of the series $\sum \frac{1}{1+n^2}$ where n is a natural number.	**4**	**CO3**
Q 5	Prove that the series $\sum \frac{1}{4^n}$ converges to 1/3.	**4**	**CO3**

SECTION B **(10x4 = 40 marks)**

Q 6	Let $S = \left\{\frac{1}{m} + \frac{1}{n} : m \in \mathbb{N},\ n \in \mathbb{N}\right\}$ a. Show that 0 is a limit point of S. b. $k \in N$, show that $\frac{1}{k}$ is a limit point of S. c. Find S' (the derived set of S).	**10**	**CO1**
Q 7	Discuss the convergence of the series $\frac{1}{3} + \frac{1.2}{3.5} + \frac{1.2.3}{3.5.7} + \frac{1.2.3.4}{3.5.7.9} + \cdots$	**10**	**CO3**
Q 8	Test the convergence of the series $\sum \frac{\sqrt{n}}{n^2+1}$.	**10**	**CO3**
Q 9	Show that the sequence $< S_n >$ where $S_1 = \frac{1}{2}, S_{n+1} = \frac{2S_n+1}{3}, n \in N$ is convergent. Also determine its limit. **OR** Let $\{x_n\}$ be a sequence defined by $x_1 = 1$, and $x_{n+1} = \sqrt{x_n^2 + \frac{1}{2^n}}$. Show that $\{x_n\}$ is convergent.	**10**	**CO2**

Section C **(20x2=40 marks)**

Q 10	a) Let S be a nonempty subset of R which is bounded above. Set $s = sup\, S$. Show that there exists a sequence $\{x_n\}$ in S which converges to s.	**10**	**CO2**

	b) Prove that each bounded sequence of real numbers has a convergent subsequence.	**10**	
Q 11	a) Test for the convergence of the series $\sum_{n=1}^{\infty}\left[\frac{1}{n}+\frac{(-1)^{n+1}}{\sqrt{n}}\right]$. b) Show that the series $\sum_{n=1}^{\infty}\frac{\sin n\theta}{n^2}$ is absolutely convergent. **OR** a) Discuss the convergence of the series $\frac{1^2}{2^2}+\frac{1^2.3^2}{2^2.4^2}+\frac{1^2.3^2.5^2}{2^2.4^2.6^2}+\cdots$ b) For all positive values of *x*, test the convergence of the series $1+\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+\cdots$	**20**	**CO3**