Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2022

Course: Finite Element Methods — **Semester: VI**

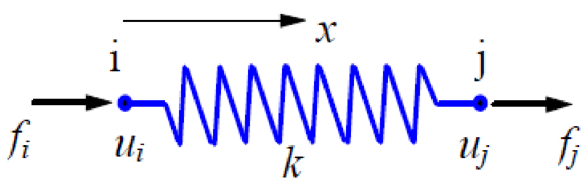
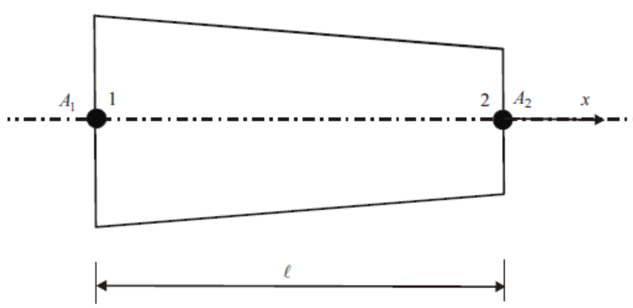
Program: B. Tech Aerospace Engineering — **Time: 03 hrs.**

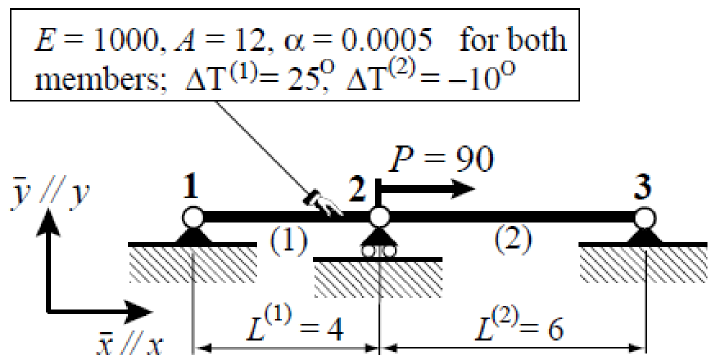
Course Code: MECH4007P — **Max. Marks: 100**

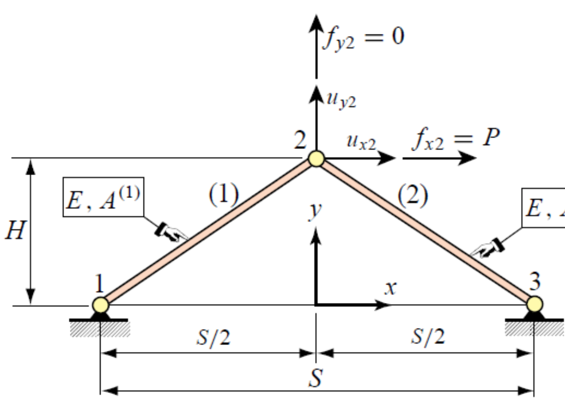
Instructions:

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Given the following stress tensor $$\sigma = \begin{bmatrix} 10 & 20 & 30 \\ 20 & 40 & 50 \\ 30 & 50 & 60 \end{bmatrix}$$ Calculate the traction vector on a plane with unit normal $\boldsymbol{n} = (0.100, 0.700, 0.707)$	**[04]**	**CO1**
Q 2	What do you mean by weak form of the differential equation? State the advantages of the weak form over the weighted residual method.	**[04]**	**CO2**
Q 3	Using matrix algebra derive, $$\bar{\mathbf{f}} = \begin{bmatrix} \bar{f}_{xi} \\ \bar{f}_{yi} \\ \bar{f}_{xj} \\ \bar{f}_{yj} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{u}_{xi} \\ \bar{u}_{yi} \\ \bar{u}_{xj} \\ \bar{u}_{yj} \end{bmatrix} = \bar{\mathbf{K}}\,\bar{\mathbf{u}},$$ from $F = k_s d = \frac{EA}{L} d$, and $F = \bar{f}_{xj} = -\bar{f}_{xi}$, $d = \bar{u}_{xj} - \bar{u}_{xi}$,	**[04]**	**CO2**
Q 4	Explain why arbitrarily oriented mechanical loads on an idealized pin-jointed truss structure must be applied at the joints. [Hint: idealized truss members have no bending resistance.] How about actual trusses: can they take loads applied between joints?	**[04]**	**CO2**

Q 5	We know that the 1st invariant of a deviatoric strain tensor is automatically zero regardless of the 1st invariant's value for the total strain tensor. But do the 2nd and 3rd invariants change between the total strain and the deviatoric strain tensors?	**[04]**	**CO1**
	SECTION B **(4Qx10M= 40 Marks)**		
Q 6	Consider a single spring element with the given notations,  Two nodes: i j Nodal displacements: u_i u_j Nodal forces: f_i f_j Spring constant (stiffness) k Using the spring-displacement relationship, derive the expression **ku** = **f** where, k = (element) stiffness matrix u = (element nodal) displacement vector f = (element nodal) force vector	**[10]**	**CO1**
Q 7	Consider a bar element whose area of cross-section varies linearly along the longitudinal axis. Derive its stiffness matrix. How will this compare with the stiffness matrix obtained assuming that the bar is of uniform cross section area equal to that of its mid-length?  	**[10]**	**CO4**

Q 8	Solve the following equation using a two-parameter trial solution by (a) the point collocation at x = 1/3 and x = 2/3; (b) the Galerkin method. $$\frac{dy}{dx} + y = 0, \qquad y(0) = 1$$	**[10]**	**CO3**
Q 9	Describe briefly the Method of Weighted Residuals (MWR). Furthermore, explain the application of MWR in the following methods, (i) Method of Point Collocation (ii) Method of Collocation by Sub-Regions **OR** Define the following terms; (i) Stiffness matrix (ii) Total potential energy (iii) Principle of minimum potential energy (iv) Shape function (v) Domain Residual	**[10]**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Two truss members are connected in series as shown in fig and fixed at the ends. Properties E = 1000, A = 12 and α = 0.0005 are common to both members. The member lengths are 4 and 6. A mechanical load P = 90 acts on the roller node. The temperature of member (1) increases by $\Delta T(1) = 25^0$ while that of member (2) drops by $\Delta T(2) = -10^0$. Find the stress in both members.  	**[20]**	**CO4**

Q 11	Derive the equivalent spring formula $F = (EA/L)\ d$ by the Theory of Elasticity relations $e = d\bar{u}(\bar{x})/d\bar{x}$ (strain-displacement equation), $\sigma = Ee$ (Hooke's law) and $F = A\sigma$ (axial force definition). Here e is the axial strain (independent of x) and σ the axial stress (also independent of x). Finally, $u(x)$ denotes the axial displacement of the cross section at a distance x from node i , which is linearly interpolated as $$\bar{u}(\bar{x}) = \bar{u}_{xi}\left(1 - \frac{\bar{x}}{L}\right) + \bar{u}_{xj}\frac{\bar{x}}{L}$$ Justify above equation is correct since the bar differential equilibrium equation: $d[A(d\sigma/d\bar{x})]/d\bar{x} = 0$, is verified for all x if A is constant along the bar. **OR** Consider the two-member arch-truss structure shown in figure. Take span $S = 8$, height $H = 3$, elastic modulus $E = 1000$, cross section areas $A^{(1)} = 2$ and $A^{(2)} = 4$, and horizontal crown force $P = f_{x2} = 12$.  Using the DSM carry out the following steps: (i) Assemble the master stiffness equations. (ii) Apply the displacement BCs and solve the reduced system for the crown displacements u_{x2} and u_{y2}. (iii) Recover the node forces at all joints including reactions. Verify that overall force equilibrium (x forces, y forces, and moments about any point) is satisfied. (iv) Recover the axial forces in the two members.	**[20]**	**CO5**