Name: Enrolment No:	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2022

Course: Ring Theory & Linear Algebra II | **Semester: VI**
Program: B. Sc. (Hons.) Maths | **Time: 03 hrs.**
Course Code: MATH 3023 | **Max. Marks: 100**

Instructions: All questions arecompulsory.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Find the number of zeros of $x^2 + 3x + 2$ in the quotient ring $\frac{\mathbb{Z}}{6\mathbb{Z}}$.	4	CO1
Q2	Prove that for any prime p, $(p-1)! \equiv -1(mod\ p)$	4	CO1
Q3	Determine whether $\mathbb{Z}[\sqrt{-5}]$ is a UFD or not. Justify your answer.	4	CO1
Q4	Find the matrix representation of the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x,y) = (2x + 3y, 3x - 2y)$ with respect to the basis $\{(1,1), (1,-1)\}$.	4	CO2
Q5	Find $trace(T)$ if $T: \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation satisfying $T^3 + I = O$ and $T \neq -I$ (where I is identity and O is null matrix in $\mathbb{R}^3$)?	4	CO2

SECTION B
(4Qx10M= 40 Marks)

Q6	Does there exists a non-constant polynomial in the ring of polynomials $\mathbb{Z}_p[x]$ (p prime) that has multiplicative inverse? Justify your answer.	10	CO1
Q7	Show that the element $1 + \sqrt{5}$ is not prime in $\mathbb{Z}[\sqrt{5}]$.	10	CO1
Q8	Find the minimal polynomial $m(t)(\deg\{m(t)\} < n)$ for the linear map $T: \mathbb{R}^n \to \mathbb{R}^n$ satisfying $T^p = I$ $(T \neq I)$ for some prime $p < n$.	10	CO2
Q9	Consider the set $S = \{(1,1,-1), (1,1,1)\} \subset \mathbb{R}^3$. Find the orthogonal complement $S^\perp$ in $\mathbb{R}^3$. Also, prove that $S^\perp$ is a subspace of $\mathbb{R}^3$. OR Prove that the vector space of all $m \times n$ matrices $M_{m,n}(\mathbb{R})$ forms an inner product space with the inner product defined as $< A, B > = trace(B^T A)$; where $A, B \in M_{m,n}(\mathbb{R})$	10	CO3

SECTION-C
(2Qx20M=40 Marks)

Q10	Consider the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x, y, z) = (z + 3x - 2y, 6y - 2x - 2z, x - 2y + 3z)$ (i) Find the minimal polynomial for T. (ii) Does there exist a T-invariant vector $X \in \mathbb{R}^3$? If yes, then find it.	**20**	**CO2**
Q11	Consider the basis $S = \{(3,1), (2,2)\}$ in the inner product space $\mathbb{R}^2$ equipped with the conventional Euclidean inner product. Normalize the vectors of S using Gram-Schmidt orthonormalizing process. OR Let $P_2(t)$ be the vector space of polynomials of degree up to 2 with standard basis $\{1, t, t^2\}$. Normalize this basis using Gram-Schmidt orthonormalizing process.	**20**	**CO3**