Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2022

Course: Ring Theory and Linear Algebra I | **Semester: IV**
Program: B.Sc. (H) Mathematics | **Time : 03 hrs.**
Course Code: MATH 2031 | **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Let a and b be arbitrary elements of a ring R whose characteristic is two and $ab = ba$. Then show that $(a+b)^2 = a^2 + b^2$.	**4**	**CO1**
Q2	If S is an ideal of a ring R with unity and $1 \in S$ then show that $S = R$.	**4**	**CO2**
Q3	Define ring homomorphism and kernel of a ring homomorphism.	**4**	**CO3**
Q4	For what value of m, the vector $(m, 3, 1)$ is a linear combination of vectors $e_1 = (3, 2, 1), e_2 = (2, 1, 0)$.	**4**	**CO4**
Q5	Determine whether or not the following vectors form a basis of R^3: $(1,1,2), (1,2,5), (5,3,4)$.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Prove that the set of matrices $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ with a, b, c integers, is a subring of the ring of 2×2 matrices having elements as integers.	**10**	**CO1**
Q7	If S_1 and S_2 are two ideals of a ring R, then show the set S_1S_2, of all elements of the form $b_1b_2 + c_1c_2 + \dots\dots\dots\dots l_1l_2$, where $b_1, c_1, \dots . l_1 \in S_1$ and $b_2, c_2, \dots . l_2 \in S_2$, is an ideal of R.	**10**	**CO2**
Q8	Let R be the ring of integers. Let R' be the set of even integers. Then show that R' is commutative ring with respect to ordinary addition and another operation $*$ in place of multiplication defined as $$a * b = \frac{ab}{2} \ \forall\, a, b \in R'$$ ab being the product of a and b under ordinary multiplication. Also, show that $R \cong R'$.	**10**	**CO3**

Q9	Prove that the union of two subspaces W_1 and W_2 is a subspace if and only if one is contained in the other. **OR** Suppose V is the vector space of all functions from R into R and V_e is the subset of all even integers $f(x)$ i.e. $f(-x) = f(x)$. Also suppose that V_o is the subset of all odd integers $f(x)$ i.e. $f(-x) = -f(x)$. Then prove that (i) V_e and V_o are subspaces of V (ii) $V_e \cap V_o = \{O\}, O$ denotes zero function.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	Let U and V be vector spaces over the field F. Let T_1 and T_2 be linear transformations from U into V. The function $T_1 + T_2$ is defined by $$(T_1 + T_2)(\alpha) = T_1(\alpha) + T_2(\alpha) \ \ for\ every\ \alpha \in U$$ is a linear transformation from U into V. If c is any element of F, the function (cT) defined by $$(cT)(\alpha) = cT(\alpha) \ \ for\ every\ \alpha \in U$$ is a linear transformation from U into V. Prove that, the set $L(U, V)$ of all linear transformations from U into V, together with the addition and scalar multiplication defined above is a vector space over the field F.	**20**	**CO5**
Q11	If S is an ideal of a ring R, then the set $\frac{R}{S} = \{S + a :\ a \in R\}$ is a ring for the two operations in $\frac{R}{S}$ defined as $$(S + a) + (S + b) = S + (a + b)$$ $$(S + a)(S + b) = S + ab \ \ \forall\, a, b \in R.$$ **OR** Two finite dimensional vector spaces over the same field are isomorphic iff they are of the same dimensions.	**20**	**CO6**