UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2022

Course: **Integral Calculus**
Program: **BSc. (Hons) Mathematics**
Course Code: **MATH1030**

Semester: **II**
Time   **: 03 hrs.**
Max. Marks: 100

Instructions:

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	For the function $f(x) = 8 - 2x$, find $c \in [0, 4]$ such that $f(c)$ is the average value of the function $f(x)$.	4	CO1
Q 2	Compute $\Gamma(-\frac{3}{2})$.	4	CO2
Q 3	Evaluate the integral $$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dxdydz$$	4	CO3
Q 4	Use double integration to find the area of a bounded region between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$	4	CO4
Q 5	Derive reduction formula for $\int \cos^n \theta \, d\theta$.	4	CO5

SECTION B
(4Qx10M= 40 Marks)

Q 6	Use differentiation under Integral sign to evaluate $$\int_0^1 \frac{x^y - 1}{\log x} dx$$	10	CO1
Q 7	State and prove the duplication formula for Gamma function.	10	CO2
Q 8	Evaluate $$\iint_R x^2 \, dxdy$$ where R is the domain in the first quadrant, bounded by lines $x = y$, $y = 0$, $x = 8$ and the curve $xy = 16$.	10	CO3
Q 9	Find the volume of the solid generated by the revolution of the *tractrix*	10	CO4

	$$x = a \cos t + \frac{1}{2} a \log \tan^2 t/2, \quad y = a \sin t$$ about its asymptotes. **OR** Find the volume of the solid obtained by revolving the ellipse $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ about the x-axis.		
	SECTION-C **(2Qx20M=40 Marks)**		
Q 10	Find the value of $$\iiint x^{l-1} y^{m-1} z^{n-1} dx\, dy\, dz$$ where x, y, z are always positive and $$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$$ Find volume of the ellipsoid $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$	20	CO3
Q11	Evaluate the definite integral, $$\int_0^{\frac{\pi}{2}} \sin^p \theta \cos^q \theta \, d\theta$$ using the reduction formula. Use the formula to evaluate $\int_0^{\frac{\pi}{2}} \sin^5 \theta \cos^6 \theta \, d\theta$ **OR** Evaluate the definite integral $$\int_0^{\frac{\pi}{2}} \cos^n \theta \, d\theta$$ using the reduction formula. Use the formula to evaluate (i) $\int_0^{\frac{\pi}{2}} \cos^7 \theta \, d\theta$ (ii) $\int_0^{\frac{\pi}{2}} \cos^8 \theta \, d\theta$	20	CO5