Name: Enrolment No:	UPES — UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2022

Course: Mathematics II — **Semester: II**
Program: B.Tech. (Non-CIT) — **Time: 03 hrs.**
Course Code: MATH 1027 — **Max. Marks: 100**

Instructions:
Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (Each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 8 and 11 have internal choice.

SECTION A

S. No.		Marks	CO
Q 1	If $x^h y^k$ is an integrating factor of the non-exact differential equation $$(2x^2y^2 + y)dx - (x^3y - 3x)dy = 0,$$ then find h and k.	**4**	**CO1**
Q 2	Find the solution of the differential equation $$2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 6y = 0 \ (x > 0).$$	**4**	**CO1**
Q 3	Determine for what value(s) of integer n the function $u(x,y) = x^n - y^n$ is harmonic?	**4**	**CO2**
Q 4	Identify the type of singularity of the function $f(z) = e^{1/z}$ at $z = 0$.	**4**	**CO3**
Q 5	Classify the partial differential equation $x^2 u_{tt} - u_{xx} + u = 0$.	**4**	**CO4**

SECTION B

S. No.		Marks	CO
Q 6	Find the general solution of the differential equation $$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 7^x - \log_e 3.$$	**10**	**CO1**
Q 7	Using Cauchy's integral formula, evaluate $\int_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz$, where C is the circle $\lvert z\rvert = 1$.	**10**	**CO2**
Q 8	Apply Residue theorem to evaluate the integral $\int_C \frac{e^z-1}{z(z-1)(z-i)^2} dz$ where C is the circle $\lvert z\rvert = 2$. **OR** Find Taylor's and Laurent's series expansion of the function $$f(z) = \frac{1}{(z+1)(z+2)^2}$$ in the regions **(i)** $\lvert z-1\rvert < 2$ **(ii)** $\lvert z-1\rvert > 3$.	**10**	**CO3**

Q 9	Form a partial differential equation by eliminating the arbitrary constants a, b, c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.	**10**	**CO4**
SECTION-C (2Qx20M=40 Marks)			
Q 10	**(i)** Evaluate the real definite integral $\int_{-\infty}^{\infty}\frac{x^2}{(1+x^2)(4+x^2)}dx$ using contour integration. **(ii)** Find the bilinear transformation which maps the points $z=0,-i,-1$ into the points $w=i,1,0$.	**10+10**	**CO3**
Q 11	**(i)** Solve the partial differential equation $$\left(D^2-2DD'+D'^2\right)z=\sin(2x+3y),$$ where $D\equiv\frac{\partial}{\partial x}$ and $D'\equiv\frac{\partial}{\partial y}$. **(ii)** If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0}=b\sin^3\left(\frac{\pi x}{l}\right)$, find the displacement $y(x,t)$. **OR** **(i)** Solve the first order partial differential equation $$xy^2p+y^3q=(zxy^2-4x^3),$$ by using Lagrange's method. **(ii)** Use the method of separation of variables to solve the equation $$4\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=3u,$$ given that $u=3e^{-x}-e^{-5x}$ when $t=0$.	**10+10**	**CO4**