# UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
## End Semester Examination, December 2022

Course: Mathematical Modeling and Graph Theory      Semester: V
Program: B.Sc H Mathematics      Time: 03 hrs.
Course Code: MATH3024D      Max. Marks: 100

**Instructions: All questions are compulsory. Q9 and Q11 have internal choice.**

## SECTION A
### (5Qx4M=20Marks)

| S. No. | | Marks | CO |
|---|---|---|---|
| Q1 | Show that $(2n + 1)xP_n(x) = (n + 1)P_{n+1}(x) + nP_{n-1}(x)$. | 4 | CO1 |
| Q2 | Evaluate the following transforms.<br><br>(a) $L\left(\frac{1}{\sqrt{t}} + t^{\frac{3}{2}}\right)$      (b) $L(e^{-2t}(t\sin 3t))$ | 4 | CO2 |
| Q3 | Write a short notes on the linear congruence method for generating pseudo random numbers. Generate 15 random numbers using $a = 1$, $b = 7$ and $c = 10$ in the formula of the linear congruence method. Was there cycling? If so, when did it occur? | 4 | CO3 |
| Q4 | Write a short notes on the sensitivity analysis. | 4 | CO5 |
| Q5 | Define the 'degree sequence' of a graph. Does there exists a simple graph with 7 vertices having degree sequence $(1,3,3,4,5,6,6)$? Justify your answer with a proper reasoning. | 4 | CO6 |

## SECTION B
### (4Qx10M= 40 Marks)

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| Q6 | State and prove the Convolution theorem. | 10 | CO2 |
| Q7 | Find the inverse Laplace transform of $\emptyset(s) = \left(\frac{5s^2 - 15s - 11}{(s+1)(s-2)^3}\right)$ | 10 | CO2 |
| Q8 | Sunrise Bakery keeps stock of popular brand of cake. Previous experience indicates the daily demand as given below<br><br>| Daily Demand | 0 | 10 | 20 | 30 | 40 | 50 |<br>|---|---|---|---|---|---|---|<br>| Probability | 0.01 | 0.20 | 0.15 | 0.5 | 0.12 | 0.02 |<br><br>Using the sequence of random numbers: $48, 78, 19, 51, 56, 77, 15, 14, 68$ and $09$, simulate the demand for the next 10 days. Find out the stock situation if the owner of the bakery decides to make 30 cakes every day. Also, estimate the average daily demand for the cakes on the basis of the simulated data. | 10 | CO3 |

| Q9 | Solve the following LPP using the simplex method.<br><br>Maximize $z = 4x + 6y$<br>Subject to $2x + 3y \leq 6$<br>$\qquad\qquad x - y \leq 1$<br>and $\qquad x, y \geq 0$<br><br>**(OR)**<br><br>Solve the following LPP Using the Graphical method.<br><br>Min $Z = 10x + 5y$<br>Subject to $0.5x + y \leq 20$<br>$\qquad\qquad 1.5x + 0.5y \geq 15$<br>$\qquad\qquad 2x + 1.5y \geq 30$<br>and $\qquad x, y \geq 0.$ | 10 | CO4 |
|---|---|---|---|

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<tr><td colspan="4" align="center"><strong>SECTION-C</strong><br><strong>(2Qx20M=40 Marks)</strong></td></tr>
<tr>
<td>Q10</td>
<td>Apply Dijkstra's algorithm on the following graph to find the shortest distance from source vertex <strong><em>a</em></strong> to remaining vertices of the graph. Also, find the shortest path from '<strong><em>a to f</em></strong>' using the backward technique.<br><br></td>
<td>20</td>
<td>CO6</td>
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<td>Q11</td>
<td>Using Frobenius method, solve in series the differential equation<br><br>$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - 4)y = 0.$$<br><br><strong>(OR)</strong><br><br>Find the series solution of $n^{th}$ order Bessel's differential equation $x^2 y'' + xy' + (x^2 - n^2)y = 0$. Also obtain the Bessel's functions $J_n(x)$ and $J_{-n}(x)$ and the complete solution in terms of Bessel's functions.<br>.</td>
<td>20</td>
<td>CO1</td>
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