Name: Enrolment No:	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Program: B. Sc.(H) Mathematics **Semester-V**
Course: Numerical Methods **Course Code: MATH-3021**
Max. Marks: 100 **Time: 03 Hours**
Instructions:

1. Section A has 5 questions. All questions are compulsory.
2. Section B has 4 questions. All questions are compulsory. Question 9 has internal choice to attempt any one.
3. Section C has 2 questions. All questions are compulsory. Question 11 has internal choice to attempt any one.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Using Newton Raphson method, find the real root of the equation $3x - \log_{10} x = 6$ correct to four significant figures.	**4**	**CO1**
Q 2	Prove the following relation where the operators have their usual meanings: $$\delta\mu \equiv \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$$	**4**	**CO2**
Q 3	Evaluate $\int_0^6 \frac{e^x}{1+x}dx$ using Simpson's 1/3rd rule with 6 subintervals.	**4**	**CO3**
Q 4	Perform one iteration of Gauss Seidal method taking zero solution to be the initial approximation to solve: $$\begin{aligned} 27x + 6y - z &= 85 \\ x + y + 54z &= 110 \\ 6x + 15y + 2z &= 72 \end{aligned}$$	**4**	**CO4**
Q 5	Using Runge-Kutta method of fourth order, solve for $y(0.1)$ taking $h = 0.1$ given that $\frac{dy}{dx} = xy + y^2, y(0) = 1$.	**4**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

Q 6	If $u = \frac{4x^2y^3}{z^4}$ and errors in x, y, z be 0.001, compute the relative maximum error in u when $x = y = z = 1$.	**10**	**CO1**
Q 7	Apply Gauss-Jordan method to solve the equations: $$\begin{aligned} 10x + y + z &= 12 \\ 2x + 10y + z &= 13 \\ x + y + 5z &= 7 \end{aligned}$$	**10**	**CO4**

Q 8	The table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface : Height (x): 100, 150, 200, 250, 300, 350, 400 Distance (y): 10.63, 13.03, 15.04, 16.81, 18.42, 19.90, 21.27 Find the value of y when x = 375 ft.	**10**	**CO2**
Q 9	A train is moving at the speed of 30 m / sec. Suddenly brakes are applied. The speed of the train per second after t seconds is given by Time (t): 0, 5, 10, 15, 20, 25, 30, 35, 40, 45 Speed (v): 30, 24, 19, 16, 13, 11, 10, 8, 7, 5 Apply Simpson's three-eight rule to determine the distance moved by train in 45 seconds. **OR** A rod is rotating in a plane. The following table gives the angle θ (radians) through which the rod has turned for various values of the time t (seconds). t: 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 θ: 0, 0.12, 0.49, 1.12, 2.02, 3.20, 4.67 Calculate the angular velocity and acceleration of the rod when t = 0.2 sec.	**10**	**CO3**

SECTION-C
(2Qx20M=40 Marks)

Q 10	Given that $\frac{dy}{dx} = \log_{10}(x+y)$ with the initial condition $y = 1$ when $x = 0$. Find y for $x = 0.2$ and $x = 0.5$ using modified Euler's method.	**20**	**CO5**
Q 11	Show that the nth divided differences $[x_0, x_1, \ldots, x_n]$ for $u_x = \frac{1}{x}$ is $\left[\frac{(-1)^n}{x_0, x_1, \ldots, x_n}\right]$. **OR** A robot arm with a rapid laser scanner is doing a quick quality check on holes drilled in a 15"X10" rectangular plate. The centers of the holes in the plate describe the path the arm needs to take, and the hole centers are located on a Cartesian coordinate system (with the origin at the bottom left corner of the plate) given by the specifications in the following table: $x(in.)$: 2.00, 4.25, 5.25, 7.81, 9.20, 10.60 $y(in.)$: 7.2, 7.1, 6.0, 5.0, 3.5, 5.0 Find the path traversed through the six points using Lagrange's method.	**20**	**CO2**