Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Analytical Geometry **Semester: III**
Program: B.Sc. (H) Mathematics & Int. B.Sc. M.Sc. Mathematics **Time : 03 hrs.**
Course Code: MATH 2047 **Max. Marks: 100**

Instructions: Read all the below mentioned instructions carefully and follow them strictly:

1) Mention Roll No. at the top of the question paper.
2) Attempt all the parts of a question at one place only.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Show that the line $4x - y = 17$ is a diameter of the circle $x^2 + y^2 - 8x + 2y = 0$.	**4**	**CO1**
Q 2	Find the pole of the line $lx + my + n = 0$ with respect to the parabola $y^2 = 4ax$.	**4**	**CO2**
Q 3	Under what condition the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ are orthogonal to each other.	**4**	**CO3**
Q 4	Derive the equation of normal at (α, β) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	**4**	**CO3**
Q 5	Calculate the length of latus rectum to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.	**4**	**CO2**
SECTION B **(4Qx10M= 40 Marks)**			
Q 6	Find the equation to the cylinder whose axis is the straight line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and the guiding curve is the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0,\ z = 0$.	**10**	**CO4**
Q 7	Obtain the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which is parallel to the plane $lx + my + nz = 0$.	**10**	**CO3**
Q 8	A circle of radius r is concentric with the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Prove that each common tangent is inclined to the axis at an angle $tan^{-1}\sqrt{\frac{r^2-b^2}{b^2-r^2}}$ and towards its length.	**10**	**CO2**

Q 9	From the point P (1, 2, 3), PN is drawn perpendicular to the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$. Find the distance of PN and co-ordinates of N. **OR** The equations to AB are $\frac{x}{2} = \frac{y}{-3} = \frac{z}{5}$ through a point P (1, 2, 3), PN is drawn perpendicular to AB, and PQ is drawn parallel to the plane $3x + 4y + 5z = 0$ to meet AB in Q. Find the equations of PN and PQ.	**10**	**CO1**
SECTION-C (2Qx20M=40 Marks)			
Q 10	Prove that the equation $2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y - 17 = 0$ represents a cone whose vertex is at (2, 2, 1).	**20**	**CO4**
Q 11	Suppose A is a point on OX and B on OY, so that the angle OAB is constant (=α). On AB as diameter a circle is described whose plane is parallel to OZ. Prove that as AB varies, the circle generates the conic $2xy - z^2 sin^2 2\alpha = 0$. **OR** A sphere of radius R passes through the origin. Show that the extremities of the diameter parallel to the x-axis lie on each of the spheres $x^2 + y^2 + z^2 \pm 2Rx = 0$.	**20**	**CO4**