Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Program Name	:	**B. Tech. SoE (Civil+FSE+SE)**			
Course Name	:	**Engineering Mathematics-I**	**Semester**	:	**I**
Course Code	:	**MATH-1050**	**Time**	:	**03 Hrs.**
Nos. of page(s)	:	**02**	**Max Marks**	:	**100**

Instructions:
Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 8 and 11 have internal choice.

SECTION A

S. No.		Marks	CO
Q 1	Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.	**4**	**CO1**
Q 2	If $u = x^2yz - 4y^2z^2 + 2xz^3$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 4u$.	**4**	**CO2**
Q 3	Evaluate the integral $\int_0^1 \int_x^{x^2} xy\, dydx$.	**4**	**CO2**
Q 4	Determine the constant b such that $\bar{A} = (bx + 4y^2z)\hat{\imath} + (x^3 \sin z - 3y)\hat{\jmath} - (e^x + 4\cos x^2y)\hat{k}$ is solenoidal.	**4**	**CO3**
Q 5	If $\phi(x, y, z) = 3x^2y - y^3z^2$, find $(grad\ \phi)$ at the point $(1, -2, 1)$.	**4**	**CO3**

SECTION B

S. No.		Marks	CO
Q 6	Examine the function $f(x, y) = 3x^2 - y^2 + x^3$ for extreme values.	**10**	**CO2**
Q 7	If $u = xyz,\ v = x^2 + y^2 + z^2,\ w = x + y + z$, then prove that $$\frac{\partial(u,v,w)}{\partial(x,y,z)} = 2(y - z)(x - z)(x - y).$$	**10**	**CO2**
Q 8	Evaluate $\iint e^{x+y} dxdy$ over the triangle bounded by $x = 0$, $y = 0$, $x + y = 1$. **OR** Change the order of integration and hence evaluate $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y}\, dydx$.	**10**	**CO2**

Q 9	The velocity vector field of an ideal fluid is given by $\bar{F}(x,y,z) = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$. Show that $\bar{F}$ is irrotational and incompressible.	**10**	**CO3**
SECTION-C			
Q 10	Define eigenvalues and eigenvectors of a matrix. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.	**20**	**CO1**
Q 11	Define curl of a vector point function. Prove that the curl of the linear velocity of any particle of a rigid body is equal to twice the angular velocity of the body. Also show that the vector field $\bar{F} = \frac{a(x\hat{\imath}+y\hat{\jmath})}{\sqrt{(x^2+y^2)}}$ is a source field or sink field according as $a > 0$ or $a < 0$. **OR** State Green's theorem. Verify Green's theorem for $$\oint_C [(x^2 - 2xy)dx + (x^2y + 3)dy]$$ where C is the boundary of the region bounded by the parabola $y = x^2$ and the line $y = x$.	**20**	**CO3**