


Name:			
Enrolment No:			
UNIVERSITY OF PETROLEUM AND ENERGY STUDIES End Semester Examination, December 2022			
Course: Differential Calculus Program: B. Sc. (Hons.) Mathematics Course Code: MATH 1044		Semester: I Time: 03 hrs. Max. Marks: 100	
SECTION A (5Qx4M=20Marks)			
Instruction: Section A has 5 questions. All questions are compulsory.			
S. No.		Marks	CO
Q 1	Suppose that $f(x)$ is a continuous in $[0,1]$ and $f(0) = 0, f(1) = 0$. Prove that $f(c) = 1 - 2c^2$ for some $c \in (0,1)$.	4	CO1
Q 2	Expand the function $\sin x$ in powers of $(x - \frac{\pi}{2})$.	4	CO1
Q 3	Show that the length of the portion of the tangent to the curve $x = a \cos^3 \theta, y = a \sin^3 \theta$ intercepted between the co-ordinate axes is constant.	4	CO2
Q 4	Show that the curvature of the point $(3a/2, 3a/2)$ on the Folium $x^3 + y^3 = 3axy$ is $-8\sqrt{2}/3a$.	4	CO3
Q 5	Evaluate the following limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 - y^3}{x^2 + y^2}$	4	CO4
SECTION B (4Qx10M= 40 Marks)			
Instruction: Section B has 4 questions. All questions are compulsory. Question 9 has internal choice to attempt any one.			
Q 6	Suppose that the angle of inclination from the top of a 100 ft pole to sun is decreasing at a rate of 0.05 radians per minute. How fast is the length of the pole's shadow on the ground increasing when the angle of inclination is $\pi/6$ radians? You may assume that the pole is perpendicular to the ground.	10	CO5
Q 7	If $y = \cos(m \sin^{-1} x)$, show that $(1 - x^2) y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.	10	CO1

Q 8	Find the condition for the curves $ax^2 + by^2 = 1, a_1x^2 + b_1y^2 = 1$ to intersect orthogonally.	10	CO2
Q 9	If $r = a(1 + \cos \theta)$, find the polar sub-tangent, polar sub-normal and the length of polar tangent and polar normal when $\theta = \tan^{-1}(3/4)$. <p style="text-align: center;">OR</p> For the curve $y = a \log \sec(x/a)$, prove that $\frac{d^2x}{ds^2} = \frac{1}{2a} \sin \frac{2x}{a}.$	10	CO2
SECTION-C (2Qx20M=40 Marks)			
Instruction: Section C has 2 questions. All questions are compulsory. Question 11 has internal choice to attempt any one.			
Q 10	(a) Given $f(x, y) = \begin{cases} \frac{(x-1) \sin y}{y \log_e x}, & (x, y) \neq (1, 0) \\ 1, & (x, y) = (1, 0) \end{cases}$ Check the continuity of the given function at point (1,0). (b) Find the value of n so that the equation $v = r^n(3 \cos^2 \theta - 1)$ satisfies the relation $\frac{\partial}{\partial r} \left(r^2 \frac{\partial v}{\partial r} \right) + \frac{1}{\sin \theta} \cdot \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v}{\partial \theta} \right) = 0.$	10+10	CO4
Q 11	Give the definition of asymptote of a curve. Also, find all the asymptotes of the following curve $y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x^2 + 2y - 2x = 1.$ <p style="text-align: center;">OR</p> Trace the curve $y^2x^2 = x^2 - a^2.$	20	CO3