



UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2021

Course: Mathematics III (Statistical and Numerical Methods)
Program: B.Tech Mechanical Engineering

Semester: III
Duration : 3 hrs

Course Code: MATH 2008

Max. Marks: 100

Instructions: Read the instructions given below carefully:

1. All questions are compulsory.
2. Statistical Tables are provided along with the question paper.

SECTION A
(Scan and Upload)

(5Q x 4M = 20 Marks)

Q.1	A bag contains four white and two black balls and a second bag contains three of each color. A bag is selected at random, and a ball is then drawn at random from the bag chosen. What is the probability that the ball drawn is white?	4	CO1
Q.2	A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.	4	CO2
Q.3	The means of two large samples of 1000 and 2000 members are 168.75 cms and 170 cms respectively. Can the samples be regarded as drawn from same population of standard deviation 6.25cms.	4	CO2
Q.4	A committee is to be formed by choosing two boys and four girls out of a group of five boys and six girls. What is the probability that a particular boy named A and a particular girl named B are selected in the committee?	4	CO1
Q.5	Ten percent of the screws produced in a certain factory turn out to be defective. Find the probability that in a sample of 10 screws chosen at random, exactly two will be defective.	4	CO1

SECTION B
(Scan and Upload)

(4Q x 10M = 40 Marks)

Q.1	If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals (a) Exactly 3 (b) More than 2 individuals (c) None (d) More than one individual, will suffer a bad reaction.	10	CO1
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Q.2	A survey of 320 families with 5 children is given below:							10	CO 2	
	No. of boys	5	4	3	2	1	0			Total
	No. of girls	0	1	2	3	4	5			
	No. of families	14	56	110	88	40	12			320
Is this result consistent with hypothesis i.e.: the male and female birth are equally possible.										

Q.3	Using Newton's Raphson Method, Find the real root of $x \log_{10} x = 1.2$ correct to five decimal places.	10	CO 3
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Q.4	Using modified Euler's method, find $y(0.2)$ given $y' = y + e^x, y(0) = 0$.	10	CO4
OR			
Solve $u_{xx} + u_{yy} = 0$, in $0 \leq x \leq 4, 0 \leq y \leq 4$ given that $u(0, y) = 0 ; u(4, y) = 8 + 2y;$ $u(x, 0) = \frac{x^2}{2}$ and $u(x, 4) = x^2$. Take $h = k = 1$ and obtain the result correct up to one decimal place.			

SECTION C
(Scan and Upload) **(2Q x 20M = 40 Marks)**

Q.1	a) Solve by Jacobi's iteration method the equation $20x + y - 2z = 17 ; 3x + 20y - z = -18; 2x - 3y + 20z = 25$	20	CO3														
	b) Use Simpson's 1/3 rd rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates.																
	OR																
	a) Find the value of $\cos(1.74)$ from the following table:																
	<table border="1" style="width: 100%; text-align: center;"> <tr> <td>x</td> <td>1.7</td> <td>1.74</td> <td>1.78</td> <td>1.82</td> <td>1.86</td> </tr> <tr> <td>$\sin x$</td> <td>0.9916</td> <td>0.9857</td> <td>0.9781</td> <td>0.9691</td> <td>0.9584</td> </tr> </table>	x	1.7	1.74	1.78	1.82	1.86	$\sin x$	0.9916	0.9857	0.9781	0.9691	0.9584				
x	1.7	1.74	1.78	1.82	1.86												
$\sin x$	0.9916	0.9857	0.9781	0.9691	0.9584												
	b) In the table below, the values of y are consecutive terms of a series of which 23.6 is the sixth term. Find the first and tenth term of the series:																
	<table border="1" style="width: 100%; text-align: center;"> <tr> <td>x</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> </tr> <tr> <td>y</td> <td>4.8</td> <td>8.4</td> <td>14.5</td> <td>23.6</td> <td>36.2</td> <td>52.8</td> <td>73.9</td> </tr> </table>	x	3	4	5	6	7	8	9	y	4.8	8.4	14.5	23.6	36.2	52.8	73.9
x	3	4	5	6	7	8	9										
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9										

Q.2	a) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$.	20	CO4
b) Solve the Poisson equation $u_{xx} + u_{yy} = -81xy, 0 < x < 1, 0 < y < 1$ given that $u(0, y) = 0, u(x, 0) = 0, u(1, y) = 100, u(x, 1) = 100$ and $h = 1/3$.			

CHI-SQUARE DISTRIBUTION TABLE

Entries provide the solution to $\Pr(\chi^2 > \chi_p^2) = p$ where χ^2 has a chi-square distribution with the indicated degrees of freedom.

df	$\chi_{0.100}^2$	$\chi_{0.050}^2$	$\chi_{0.025}^2$	$\chi_{0.010}^2$	$\chi_{0.005}^2$
1	2.706	3.841	5.024	6.635	7.879
2	4.605	5.991	7.378	9.210	10.597
3	6.251	7.815	9.348	11.345	12.838
4	7.779	9.488	11.143	13.277	14.860
5	9.236	11.070	12.833	15.086	16.750
6	10.645	12.592	14.449	16.812	18.548
7	12.017	14.067	16.013	18.475	20.278
8	13.362	15.507	17.535	20.090	21.955
9	14.684	16.919	19.023	21.666	23.589
10	15.987	18.307	20.483	23.209	25.188
11	17.275	19.675	21.920	24.725	26.757
12	18.549	21.026	23.337	26.217	28.300
13	19.812	22.362	24.736	27.688	29.819
14	21.064	23.685	26.119	29.141	31.319
15	22.307	24.996	27.488	30.578	32.801
16	23.542	26.296	28.845	32.000	34.267
17	24.769	27.587	30.191	33.409	35.718
18	25.989	28.869	31.526	34.805	37.156
19	27.204	30.144	32.852	36.191	38.582
20	28.412	31.410	34.170	37.566	39.997
21	29.615	32.671	35.479	38.932	41.401
22	30.813	33.924	36.781	40.289	42.796
23	32.007	35.172	38.076	41.638	44.181
24	33.196	36.415	39.364	42.980	45.559
25	34.382	37.652	40.646	44.314	46.928
26	35.563	38.885	41.923	45.642	48.290
27	36.741	40.113	43.195	46.963	49.645
28	37.916	41.337	44.461	48.278	50.993
29	39.087	42.557	45.722	49.588	52.336
30	40.256	43.773	46.979	50.892	53.672
40	51.805	55.758	59.342	63.691	66.766
50	63.167	67.505	71.420	76.154	79.490
60	74.397	79.082	83.298	88.379	91.952
70	85.527	90.531	95.023	100.425	104.215
80	96.578	101.879	106.629	112.329	116.321
90	107.565	113.145	118.136	124.116	128.299
100	118.498	124.342	129.561	135.807	140.169