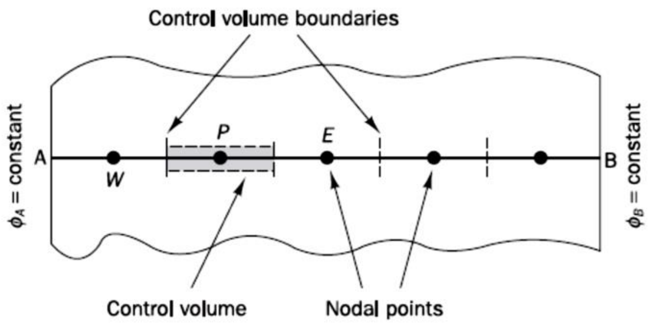
UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, January 2022

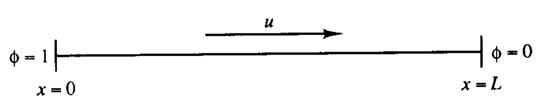
Course: Finite Volume Methods for Conservation Laws | **Semester: I**
Program: M. Tech CFD | **Time: 03 hrs.**
Course Code: ASEG 7021 | **Max. Marks: 100**
Pages: 04
Instructions: Make use of sketch/plots to elaborate your answer. All sections are compulsory

SECTION A
(Scan and upload) **(5Qx 4M = 20 Marks)**
Give appropriate justification for your choice

S. No.		**Marks**	**CO**
Q 1	Finite volume discretization equation for scalar variable $\emptyset$ is obtained as: $$-5\emptyset_P = -3\emptyset_E - 2\emptyset_W + 5$$ Is the above discretization expected to yield a physically unrealistic solution? Justify with reasoning.	**[04]**	**CO2**
Q 2	Which of these terms need a surface integral? a) Diffusion and rate of change terms b) Convection and source terms c) Convection and diffusion terms d) Diffusion and source terms	**[04]**	**CO1**
Q 3	Which of these terms need a volume integral while modelling steady flows? a) Convection term b) Diffusion term c) Source term d) Rate of change term	**[04]**	**CO1**
Q 4	For a 1-D convection – diffusion problem, fluid density = 1000 kg/m^3, flow velocity = 1 *m/s*, diffusion coefficient = 10^{-9} m^2/s, and domain length = 1 *m*. Will a central difference scheme work, for a numerical solution of this problem (Given that dimension of the solution vector for the TDMA should not exceed 1000)? Give reasons for your answer.	**[04]**	**CO2**
Q 5	The discretization of the transient term using the finite volume approach is more like the spatial discretization of ___________ a) the convection term b) the diffusion term c) the source term d) the anti-diffusion term	**[04]**	**CO3**

SECTION B (Scan and upload) (4Qx10M = 40 Marks)			
Q 6	Explain in details the philosophy of the SIMPLE method. What is the need for a staggered grid?	**[10]**	**CO2**
Q 7	Consider the steady state diffusion of a property ϕ in a one-dimensional domain defined in figure. The process is governed by $$\frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) + S = 0$$ where Γ is the diffusion coefficient and S is the source term. Boundary values of ϕ at points A and B are prescribed.  Explain the several steps involved in discretizing the geometry and the equation to obtain the appropriate solutions of the governing differential equation.	**[10]**	**CO3**
Q 8	In compressible viscous flows the energy equation is completely decoupled from the continuity and momentum equations, i.e. the solution of energy equation is not required for obtaining pressure and velocity fields. Prove it.	**[10]**	**CO3**
Q 9	Classify the steady two-dimensional velocity potential equation: $$\left(1 - M^2\right)\phi_{xx} + \phi_{yy} = 0$$ where M is mach number. Explain the physical meaning of various classifications based on M. **OR** What do you mean by initial and boundary conditions? Define various types of boundary conditions which are usually encountered in CFD problems.	**[10]**	**CO4**

SECTION-C
(Scan and upload) **(2Qx 20M= 40 Marks)**

Q 10	Consider a 1-D steady state convection – diffusion problem without any source term. Derive a profile assumption for variation of the dependent variable in the advection term, following the QUICK scheme. Derive the complete disctretization equation for the convection-diffusion problem. Assess your discretization in perspective of the basic rule regarding the sign of coefficients of the discretized equation. Extend your derivations made to a 1 – D unsteady state convection-diffusion problem with fully explicit time discretization.	**[20]**	**CO5**
Q 11	A property $\emptyset$ is transported by means of convection and diffusion through the one-dimensional domain sketched in figure below. The governing equation below; $$\frac{d}{dx}(\rho u\emptyset) = \frac{d}{dx}\left(\tau\frac{d\emptyset}{dx}\right)$$ boundary conditions are $\emptyset_0 = 1$ at $x = 0$ and $\emptyset_L = 0$ at $x = L$. Using five equally spaced cells (for first two cases) and the central differencing scheme for convection and diffusion calculate the distribution of $\emptyset$ as a function of x for cases: (i) Case 1: u = 0.1 *m/s*, using 5 cells (ii) Case 2: u = 2.5 *m/s*, using 5 cells (iii) Case 3: u = 2.5 *m/s*, using 10 cells  and compare the results with the analytical solution given below. The following data apply: length $L = 1.0$ *m*, p = 1.0 *kg/m*^{3}, $\Gamma = 0.1$ *kg/m/s*. $$\frac{\phi - \phi_0}{\phi_L - \phi_0} = \frac{\exp(\rho ux/\Gamma) - 1}{\exp(\rho uL/\Gamma) - 1}$$ **OR** In a steady two-dimensional situation, the variable ϕ is governed by $$div\,(\rho\boldsymbol{u}\phi) = div(\Gamma grad\phi) + a - b\phi$$ where $\rho = 1$, $\Gamma = 1$, $a = 10$, and $b = 2$. The flow field is such that u = 1 and v = 4	**[20]**	**CO5**

everywhere. For the uniform grid shown in the figure $\Delta x = \Delta y = 1$. The values of ϕ are given for the boundaries. Adopting the control volume design calculate the values of $\phi_1, \phi_2, \phi_3,$ and ϕ_4 by the use of:

(a) The central-difference scheme

(b) The upwind scheme

(c) The hybrid scheme

(d) The power-law scheme