UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

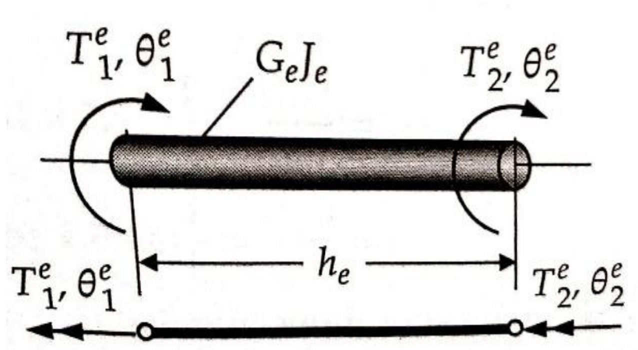
End Semester Examination, May 2021

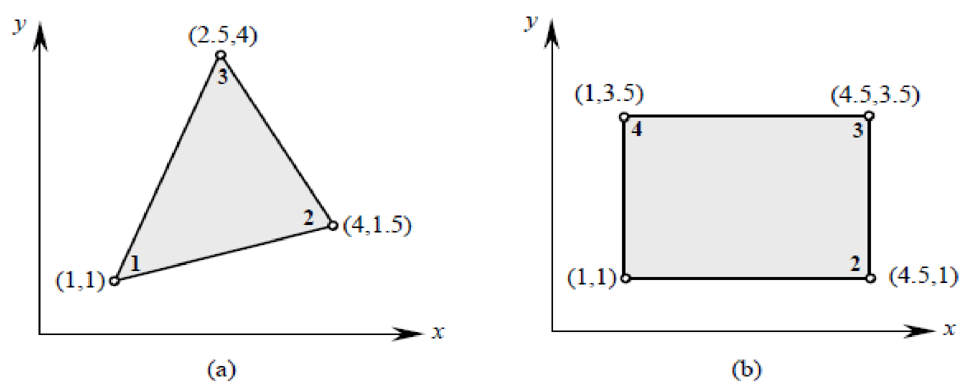
Programme: B.Sc. (Hons.) Mathematics | **Semester: VI**
Course Name: Finite Element Methods | **Max. Marks: 100**
Course Code: MATH 3027 | **Duration: 3 Hrs.**
No. of page/s: 04

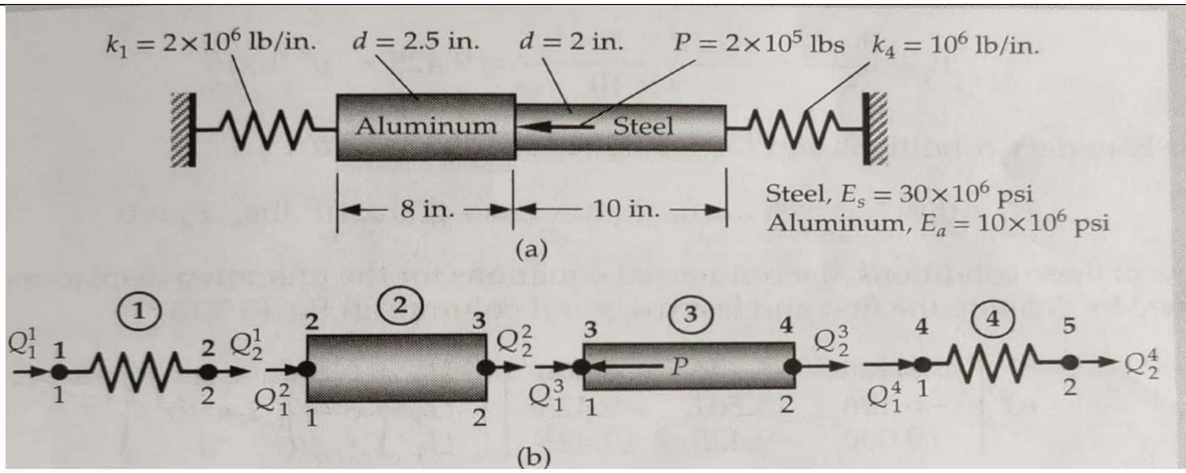
Section A

Attempt all the questions. This section contains 6 multiple-choice questions and one option is correct. Write the correct option. Each question carries 5 marks.

1.	Consider the approximate linear polynomial $u_h^e(x,y) = c_1^e + c_2^e x + c_3^e y$ in x and y in Ω_e . In the Finite Element Method, triangular elements with narrow geometries should be avoided since: A. Any two of three nodes are very close to each other or three nodes almost on a line, the coefficient matrix can be nearly singular and numerically non-invertible. B. We can consider two of three nodes very close to each other since theoretically, the matrix will not be singular. C. Narrow geometries do not have any impact on finite element meshes since there is a systematic procedure to obtain interpolation functions for triangular meshes. D. We should avoid only singular matrix since nearly singular matrix does not affect the solution using finite element method	**CO5**
2.	In the Least Square Method, let $u(x) \approx u_2(x) = \sum_{j=1}^{2} c_j\varphi_j + \varphi_0(x)$ be the approximation of $u(x)$ in the two-parameter solution of the following differential equation: $\frac{d^2y}{dx^2} + y = x^2,\ y(0) = 0,\ \left[\frac{dy}{dx}\right]_{x=1} = 1$. Which of the following is correct? A. $\varphi_1 = x(2+x),\ \varphi_2 = x^2\left(1-\frac{3}{2}x\right), \varphi_0 = 1$ B. $\varphi_1 = x\left(2-\frac{3}{2}x\right),\ \varphi_2 = x^2(1-x), \varphi_0 = x^2$ C. $\varphi_1 = x\left(2-\frac{2}{3}x\right),\ \varphi_2 = x^2(1-x), \varphi_0 = 0$ D. $\varphi_1 = x(2-x),\ \varphi_2 = x^2\left(1-\frac{2}{3}x\right), \varphi_0 = x$	**CO1**
3.	In the Galerkin method, let $u(x) \approx u_1(x) = c_1x(1-x) + (1-x)$ be the approximation of $u(x)$ in one parameter solution of the following differential equation: $-2u\frac{d^2u}{dx^2} + \left(\frac{du}{dx}\right)^2 = 4,\ u(0) = 1,\ u(1) = 1.$ Then the residual R is: A. $R = -3 + 2c_1 + c_1^2$ B. $R = -3 - 2c_1 + c_1^2$ C. $R = -3 + 2c_1 + c_1^2 + 4c_1x$ D. $= -3 + 2c_1 + c_1^2 - 4c_1x$	**CO2**

4.	In the mechanics of deformable solids, the angle of the twist θ of an elastic, constant cross-section, circular cylindrical member is related to torque T by $T = k\theta$ where $k = \frac{GJ}{L}$. Here J denotes the polar moment area, L is the length, and G is the shear modulus of the material of the shaft. Then the relationship between the end torques (T_1^e, T_2^e) and end twists (θ_1^e, θ_2^e) of the torsional dinite element (shown in the figure) is:  A. $\begin{bmatrix} T_1^e \\ T_2^e \end{bmatrix} = \begin{bmatrix} k_e & k_e \\ k_e & k_e \end{bmatrix} \begin{bmatrix} \theta_1^e \\ \theta_2^e \end{bmatrix}$ B. $\begin{bmatrix} T_1^e \\ T_2^e \end{bmatrix} = \begin{bmatrix} -k_e & k_e \\ k_e & -k_e \end{bmatrix} \begin{bmatrix} \theta_1^e \\ \theta_2^e \end{bmatrix}$ C. $\begin{bmatrix} T_1^e \\ T_2^e \end{bmatrix} = \begin{bmatrix} k_e & -k_e \\ -k_e & k_e \end{bmatrix} \begin{bmatrix} \theta_1^e \\ \theta_2^e \end{bmatrix}$ D. $\begin{bmatrix} T_1^e \\ T_2^e \end{bmatrix} = \begin{bmatrix} -k_e & -k_e \\ -k_e & -k_e \end{bmatrix} \begin{bmatrix} \theta_1^e \\ \theta_2^e \end{bmatrix}$	**CO3**
5.	We consider the condensed finite element equations of the eigenvalue problem for the undamped system in the following general form: $(\boldsymbol{K_c} - \lambda \boldsymbol{M_c})\boldsymbol{U_c} = 0.$ Then which of the following statement is true: A. The matrices $\boldsymbol{K_c}$ and $\boldsymbol{M_c}$ are not real. B. The matrices $\boldsymbol{K_c}$ and $\boldsymbol{M_c}$ are not symmetric. C. The matrix $\boldsymbol{M_c}$ is nonsingular. D. The eigenvectors of two different eigenvalues are not orthogonal.	**CO4**
6.	If $\psi_1 = \left(1 - \frac{\bar{x}}{a}\right)\left(1 - \frac{\bar{y}}{b}\right)$, $\psi_2 = \frac{\bar{x}}{a}\left(1 - \frac{\bar{y}}{b}\right)$, $\psi_3 = \frac{\bar{x}}{a}\frac{\bar{y}}{b}$ and $\psi_4 = \left(1 - \frac{\bar{x}}{a}\right)\frac{\bar{y}}{b}$ are the linear interpolation functions for rectangular elements. Then the value of S_{11}^{12} is: A. $-\frac{1}{4}$ B. $\frac{1}{4}$ C. $\frac{b}{3a}$ D. $-\frac{b}{3a}$	**CO4**

SECTION B **Attempt all the questions. This section contains descriptive type's questions. Each question carries 10 marks.**		
7.	Construct the weak form of the following differential equation: $-\frac{d}{dx}\left(u\frac{du}{dx}\right)+f=0$ for $0<x<1$; $\left(u\frac{du}{dx}\right)_{x=0}=0,\ u(1)=\sqrt{2}$ If w is considered weight function then find $B(w,u)$. Is $B(w,u)$ symmetric and linear?	**CO1**
8.	The diffusion equation in dimensionless form is given as: $\frac{\partial^2 T}{\partial x^2}-\frac{\partial T}{\partial t}=0$ with $T(0,t)=0,\ T(1,t)=0$ The complete solution of the time-dependent problem is a linear combination of the mode Shapes U_n and temporal term $e^{-\lambda_n t}$: $T(x,t)=\sum_{n=1}^{\infty}k_nU_n(x)e^{-\lambda_n t}$ where k_n is constant. Use finite element method for the mesh of two linear elements to determine the eigenvalues and the eigenfunctions and compare the result with those obtained with the single quadratic element.	**CO5**
9.	What is the importance of natural coordinate ξ? Derive the linear, quadratic, and cubic interpolation functions for one-dimensional elements in terms of natural coordinate.	**CO4**
10.	Calculate the linear interpolation functions for linear and rectangular elements shown in the following figures:  	**CO5**
11.	Consider a stepped bar supported by springs on both ends as shown in the following figure. Using the finite element method, determine the displacements in the springs and stresses in each portion of the stepped bar. Neglect the weight of the bar and assume that the bar experiences only axial displacements.	**CO3**



SECTION C

This section contains descriptive type's question and it has internal choices. This question carries 20 marks.

12.

Use the finite element method to solve the problem described by the following differential equation and boundary conditions:

$$\frac{d^2u}{dx^2} + u = x^2 \text{ for } 0 < x < 1; \quad u(0) = 0, \left[\frac{du}{dx}\right]_{x=1} = 1.$$

Use the uniform mesh of three linear elements.

Note: Obtain the condensed equations only.

OR

Evaluate the following coefficient matrices and source vector using the linear Lagrange interporlation functions:

$$K_{ij}^e = \int_{x_a}^{x_b} (a_0^e + a_1^e x) \frac{d\psi_i^e}{dx} \frac{d\psi_j^e}{dx} dx, \; M_{ij}^e = \int_{x_a}^{x_b} (c_0^e + c_1^e x) \psi_i^e \psi_j^e dx, \; f_i^e = \int_{x_a}^{x_b} (f_0^e + f_1^e x) \psi_i^e dx$$

Where a_0^e, a_1^e, c_0^e, c_1^e, f_0^e and f_1^e are constants.

CO3