Name:

Enrolment No:

UPES
UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
END SEM Examinations (Online Mode), May 2021

Course: PDE and system of ODE **Semester:** IV
Program: B.Sc. (Hons.) Mathematics **Time:** 3 Hrs.
Course Code: MATH 2030 **Max. Marks:** 100

SECTION - A **6 x 5 = 30 Marks**

1. Each Question will carry 5 Marks
2. Instruction: Select the correct option.

Q 1	The partial differential equation $u_{xxx} + u\,u_x + u_t = 0$ is **A.** Semi – Linear, homogeneous and third order **B.** Linear, non-homogeneous and third order **C.** Quasi Linear, homogeneous and second order **D.** Nonlinear, homogeneous and third order	CO1
Q 2	The solution of the partial differential equation $u_{xx} + u_{yy} = 0$ is (are) **A.** $u(x,y) = x^2 - y^2$ **B.** $u(x,y) = e^x \sin y$ **C.** $u(x,y) = 2xy$ **D.** None of these	CO2
Q 3	The characteristic curves of the equation $x^2u_{xx} - y^2u_{yy} = x^2y^2 + x,\ x > 0$ are **A.** Rectangular hyperbola **B.** parabola **C.** circle **D.** None of these	CO3
Q 4	The PDE $y^3u_{xx} - (x^2 - 1)u_{yy} = 0$ is **A.** Parabolic in $\{(x,y): x < 0\}$ **B.** Hyperbolic in $\{(x,y): y > 0,\ x > 1\}$ **C.** Elliptic in $\mathbb{R}^2$ **D.** Parabolic in $\{(x,y): x > 0\}$	CO4
Q 5	Let $u(x,t)$ be the solution to the initial value problem $u_{tt} = u_{xx}$ for $-\infty < x < \infty, t > 0$ with $u(x,0) = \sin x, u_t(x,0) = \cos x$, then the value of $u\left(\frac{\pi}{2}, \frac{\pi}{6}\right)$ is **A.** $\frac{\sqrt{3}}{2}$ **B.** $\frac{1}{2}$ **C.** $\frac{1}{\sqrt{2}}$ **D.** 1	CO4
Q 6	The approximate values of $x(1)$ and $y(1)$ by using Picard's first approximation method for the solution of $\frac{dx}{dt} = y + t,\ \frac{dy}{dt} = t - x^2$ given that $x(0)=2$ and $y(0)=1$ are **A.** 3.5 and 2.5 **B.** 3.5 and - 2.5 **C.** - 3.5 and 2.5 **D.** -3.5 and -2.5, respectively.	CO4

SECTION – B **10 x 5 = 50 Marks**

1. Each question will carry 10 marks
2. Instruction: Answer on a separate white sheet, upload the solution as image.

Q 7	Determine the general solution of the first order PDE $x^2u_x + y^2u_y = (x + y)\,u$.	CO1
Q 8	Reduce the following equation to canonical form $x^2\,u_{xx} + 2xy\,u_{xy} + y^2\,u_{yy} = 0$.	CO2

Q 9	Determine the solution of the non-homogeneous partial differential equation $u_{xx} - u_{yy} = 1$, with $u(x,0) = \sin x, \quad u_y(x,0) = x.$	CO3
Q10	**(*a*)** Show that $x = 2e^{2t}, \ y = -3e^{2t}$, and $x = e^{7t}, \ y = e^{7t}$, are the solutions of the homogeneous linear system $\frac{dx}{dt} = 5x + 2y, \quad \frac{dy}{dt} = 3x + 4y,$ **(*b*)** Show that the two solutions defined in part (*a*) are linearly independent on every interval $a \le t \le b$, and write the general solution of the homogeneous system of part (***a***). **(*c*)** Show that $x = t + 1, \ y = -5t - 2$, is a particular solution of the nonhomogeneous linear system $\frac{dx}{dt} = 5x + 2y + 5t, \quad \frac{dy}{dt} = 3x + 4y + 17t$, and write the general solution of this system.	CO4
Q 11	Using Runge-Kutta's fourth order method, determine the approximate values of x and y corresponding to $t = 0.1$ and $t = 0.2$ given that $x(0) = 1$ and $y(0) = -1$ for $\frac{dx}{dt} = xy + t, \ \frac{dy}{dt} = yt + x.$	CO4
Section – C **1 x 20 = 20 Marks** **1. Each Question carries 20 Marks.** **2. Instruction: Answer on a separate white sheet, upload the solution as image.**		
Q 12	Determine the solution of initial boundary-value problem $u_{tt} = 9\,u_{xx}, \ \ 0 < x < \infty,$ $t > 0$, with $u(x,0) = 0, \quad 0 \le x < \infty, \quad u_t(x,0) = x^3, \quad 0 \le x < \infty, \ u_x(0,t) = 0, \ t \ge 0.$ **OR** Determine the solution of initial boundary-value problem $u_{tt} = 4\,u_{xx}, \quad 0 < x < 1, \quad t > 0,$ with $u(x,0) = 0, \quad 0 \le x < 1, \quad u_t(x,0) = x(1-x), \quad 0 \le x < 1, \ u(0,t) = 0 = u(1,t), \ t \ge 0.$	CO3