Name:
Enrollment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, June 2021

Programme Name: B.Sc Physics, Chemistry, Geology **Semester : II**
Course Name : Calculus **Time : 03 hrs**
Course Code: MATH 1033G **Max. Marks : 100**

Section A **(All questions are compulsory.)**			
1.	Consider the following function. $$f(x)=\begin{cases}2^{1/x}, & x\neq 0\\ 0, & x=0\end{cases}$$ Write if the function $f(x)$ is continuous at $x=0$.	**[5]**	**CO1**
2.	If $u=\log\frac{x^2+y^2}{x+y}$, the write the values of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$	**[5]**	**CO2**
3.	Write the sum of the intercepts of the tangent to $\sqrt{x}+\sqrt{y}=\sqrt{a}$ upon the co-ordinate axes.	**[5]**	**CO3**
4.	Write equation of an asymptote of the curve $x^3+y^3=3axy$	**[5]**	**CO4**
5.	Consider the function $x^3+y^3-3x-12y+10$. Write if the function has minimum or the maximum value at $(-1,-2)$. If the function has optimum write the optimal value at $(-1,-2)$.	**[5]**	**CO5**
6.	Write the limiting value of $\lim_{x\to 0}\frac{1}{x}-\cot x$	**[5]**	**CO6**
SECTION B **(Q1-Q5 are compulsory and Q5 has an internal choice.)**			
1.	If $y=\frac{b+cx}{a+2bx+cx^2}$ show that $$y_n=(-1)^n n!\left(\frac{c}{a+2bx+cx^2}\right)^{\frac{n+1}{2}}\times\left\{\cos(n+1)\tan^{-1}\frac{\sqrt{ac-b^2}}{b+cx}\right\}$$	**[10]**	**CO1**
2.	If $r^2=x^2+y^2+z^2$, show that $$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)r=\frac{2}{r}$$	**[10]**	**CO2**

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3.	Find the radius of curvature at any point of the curve $x = a(\theta + \cos\theta),\quad y = a(1 - \cos\theta).$	**[10]**	**CO3**
4.	Determine the position and nature of the double points on the curve $y(y-6) = x^2(x-2)^3 - 9$	**[10]**	**CO4**
5.	Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$ **OR** Trace the curve $r = a(1 + \cos\theta)$	**[10]**	**CO5**
	SECTION C **(Q1 is compulsory and has an internal choice.)**		
1A	Use Cauchy's mean value theorem to evaluate $\lim_{x\to 1}\left[\frac{\cos\frac{1}{2}\pi x}{\log\frac{1}{x}}\right]$ **OR** Use Lagrange's mean value theorem to prove that $1 + x < e^x < 1 + xe^x$	**[10]**	**CO6**
1B	Apply Maclaurin's theorem to obtain the expansion of $\sec x$. **OR** Evaluate $\lim_{x\to 0}\frac{(1+x)^{1/x} - e}{x}$	**[10]**	