UPES SAP ID No.: ________________

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
Examination, July 2020

Programme: B.Tech ASE / ASE+AVE **Semester : VI**
Course Name: Applied Numerical Methods **Max. Marks : 100**
Course Code: MATH3001 **Attempt Duration : 3 Hrs.**
No. of page/s:8

Note:

1. Read the instruction carefully before attempting.
2. This question paper has two section, Section A and Section B.
3. There are total of six questions in this question paper. **One** in **Section A** and **five** in **Section B**
4. **Section A** consist of multiple choice based questions and has the total weightage of 60%.
5. **Section B** consist of long answer based questions and has the total weightage of 40%.
6. The maximum time allocated to **Section A** is 90 minutes.
7. **Section B** to be submitted within 24 hrs from the scheduled time i.e. if the examination starts at 10:00 AM, the long answers must be submitted by 09:59:59 AM next day. Similarly, if the examination starts at 2:00 PM it must be submitted by 01:59:59 PM next day. (*Exceptional provision due extraordinary circumstance due to COVID-19 and due to internet connectivity issues in the far-flung areas*).
8. No submission of **Section B** shall be entertained after 24 Hrs.
9. **Section B** should be attempted after **Section A**
10. **The section B** should be attempted in blank white sheets (hand written) with all the details like programme, semester, course name, course code, name of the student, Sapid at the top (as in the format) and signature at the bottom (right hand side bottom corner)
11. Both section A & B should have questions from entire syllabus.
12. The COs mapping, internal choices within a section is same as earlier

Section – A (Attempt all the questions)

1. Answer all the questions

<table>
<tr><td>(a)</td><td>If $\tilde{x}$ is an approximation of exact value x, then absolute error is defined as
(a) $|x-\tilde{x}|$
(b) $x-\tilde{x}$
(c) $\frac{x-\tilde{x}}{x}$
(d) None of these</td><td>2M
CO1</td></tr>
<tr><td>(b)</td><td>An approximate value of π is given by 3.142647 and its true value is 3.1415926. Then the relative error is given by
(a) 0.000335625
(b) 0.00335625
(c) 0.0335625
(d) None of these</td><td>2M
CO1</td></tr>
<tr><td>(c)</td><td>Which of the following methods are guaranteed convergence methods? (Select all that apply)
(a) Bisection method
(b) Newton-Raphson method
(c) Fixed point iteration method
(d) False position method</td><td>2M
CO1</td></tr>
<tr><td>(d)</td><td>Without applying any interpolation formula, the degree of the polynomial governing the following data is:
<table>
<tr><td>x</td><td>0</td><td>4</td><td>8</td><td>12</td><td>16</td></tr>
<tr><td>y</td><td>6</td><td>38</td><td>390</td><td>1446</td><td>3590</td></tr>
</table>
(a) 2
(b) 3
(c) 4
(d) None of these</td><td>2M
CO2</td></tr>
<tr><td>(e)</td><td>The value of $\Delta^4 y(1)$ from the following table is
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y</td><td>2</td><td>5</td><td>10</td><td>20</td><td>30</td></tr>
</table>
(a) -8</td><td>2M
CO2</td></tr>
</table>

(b) -3
(c) -2
(d) -1

(f) If the function $y = 2x^3 - 3x^2 + 3x - 10$ is expressed in its factorial notation as $A[x]^3 + B[x]^2 + C[x] + D$, then the value of B is

2M
CO2

(a) 3
(b) -10
(c) 2
(d) None of these

(g) Consider the following table

2M
CO3

x	x_0	x_1	x_2	x_3
y	y_0	y_1	y_2	y_3

While finding $\frac{dy}{dx}$ at the tabulated point x_3 using the first derivative formula derived from Newton-Gregory backward interpolation, the coefficient of $\nabla^3 y_3$ is

(a) $\frac{1}{2h}$
(b) $\frac{1}{3h}$
(c) $-\frac{1}{3h}$
(d) None of these

(h) Using the Trapezoidal rule, the value of $\int_2^8 y \ dx$ is

2M
CO3

x	2	4	6	8
y	3	5	6	7

(a) 18
(b) 25
(c) 16
(d) 32

(i) Which of the following methods can be used to find the approximate value of $\int_0^2 e^{x^2} \sin 2x \ dx$ by dividing the interval into 10 subintervals? (Select all that apply)

2M
CO3

(a) Trapezoidal rule
(b) Simpson's 1/3 rule

	(c) Simpson's 3/8 rule (d) All the above	
(j)	Gauss elimination technique is the combination of (a) Forward elimination and Backward substitution (b) Backward elimination and Forward substitution (c) Both (a) and (b) (d) None of these	**2M** CO4
(k)	Which of the following is a diagonally dominant system? (a) $3x - 4y - z = 40; x - 2y + 12z = -86; x - 6y + 2z = -32$ (b) $4x = 2y - z - 1; x + z = -4; 3x - 5y + z = 3$ (c) $27x + 6y - z = 85; 6x + 15y + 2z = 72; x + y + 54z = 110$ (d) None of these	**2M** CO4
(l)	While decomposing the matrix $A = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}$ as a product LU where $L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$ and $U = \begin{bmatrix} p & q & r \\ 0 & s & t \\ 0 & 0 & u \end{bmatrix}$, then the values of a, b and c are (a) $a = \frac{2}{3}, b = 1, c = \frac{6}{5}$ (b) $a = 1, b = \frac{2}{3}, c = \frac{6}{5}$ (c) $a = \frac{6}{5}, b = \frac{2}{3}, c = 1$ (d) None of these	**2M** CO4
(m)	Which of the following method is a Predictor – Corrector method? (a) Taylor's method (b) Euler's method (c) Modified Euler's method (d) None of these	**2M** CO5
(n)	Given $3\frac{dy}{dx} + 5y^2 = \sin x, y(0.3) = 5$. Using a step size $h = 0.3$, the value of $y(0.9)$ using Euler's method is most nearly (a) -35.318 (b) -36.458 (c) -600.213 (d) None of these	**2M** CO5
(o)	The partial differential equation $(1 + x^2)u_{xx} + (5 + 2x^2)u_{xt} + (4 + x^2)u_{tt} = 0$ is classified as	**2M** CO6

<table>
<tr><td></td><td>(a) Parabolic
(b) Hyperbolic
(c) Elliptic
(d) None of these</td><td></td></tr>
<tr><td>(p)</td><td>The approximate value of a root of the equation $x - \sin x - 1 = 0$ using fixed point iteration technique in the interval [1 2] is

(a) 1.1345
(b) 1.9345
(c) 1.6279
(d) None of these</td><td>3M
CO1</td></tr>
<tr><td>(q)</td><td>The approximated value of a root of $x^2 + 4\sin x = 0$ in the interval $[-2 \quad -1]$ correct to three decimal places using Newton-Raphson method? -1.4567

(a) -2.4332
(b) -2.0177
(c) -1.9337
(d) None of these</td><td>3M
CO1</td></tr>
<tr><td>(r)</td><td>If y_x is a polynomial for which fifth difference is constant and $y_1 + y_7 = -7845$, $y_2 + y_6 = 686$, $y_3 + y_5$=1088, then the value of y_4 is

(a) 476
(b) 571
(c) 662
(d) None of these</td><td>3M
CO2</td></tr>
<tr><td>(s)</td><td>The value of $f'(2)$ using an appropriate interpolation formula from the following data is
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>5</td></tr><tr><td>y</td><td>2</td><td>3</td><td>12</td><td>147</td></tr></table>
(a) 2
(b) 12
(c) 15
(d) 21</td><td>3M
CO2</td></tr>
</table>

(t)	The following boundary value problem is solved using finite difference method by taking number of subintervals $n = 3$ $x\frac{d^2y}{dx^2} + y = 0; \quad y(1) = 1,\ y(2) = 2$ Then, the values of $y\left(\frac{4}{3}\right)$ and $y\left(\frac{5}{3}\right)$ are (a) $y\left(\frac{4}{3}\right) = \frac{408}{487}\ ,\ y\left(\frac{5}{3}\right) = \frac{570}{487}$ (b) $y\left(\frac{4}{3}\right) = \frac{508}{487}\ ,\ y\left(\frac{5}{3}\right) = \frac{670}{487}$ (c) $y\left(\frac{4}{3}\right) = \frac{608}{487}\ ,\ y\left(\frac{5}{3}\right) = \frac{770}{487}$ (d) $y\left(\frac{4}{3}\right) = \frac{708}{487}\ ,\ y\left(\frac{5}{3}\right) = \frac{870}{487}$	**3M** CO6
(u)	While solving the parabolic equation $u_{xx} = 2u_t, u(0,t) = u(4,t) = 0$ and $u(x,0) = x(4 - x)$ with step size $h = 1$, The values of $u(1,4)$ and $u(3,5)$obtained by Bendre-Schmidt recurrence relation are given by (a) $u(1,4) = 1.5$ and $u(3,5) = 0.75$ (b) $u(1,4) = 0.75$ and $u(3,5) = 0.5$ (c) $u(1,4) = 0.25$ and $u(3,5) = 0.95$ (d) None of these	**3M** CO6
(v)	Given the differential equation $2\frac{dy}{dx} = (1 + x^2)y^2$ and $y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21.$ The approximate value of $y(0.4)$ using Milne's predictor corrector method is (a) 1.27 (b) 1.37 (c) 1.41 (d) None of these	**3M** CO5
(w)	While solving the system $\begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ -18 \\ 25 \end{bmatrix}$ using Gauss-seidel iteration with initial values $\{x^{(0)}, y^{(0)}, z^{(0)}\} = \{0,0,0\}$, the	**3M** CO4

<table>
<tr>
<td></td>
<td>solutions obtained in the second iteration $\{x^{(2)}, y^{(2)}, z^{(2)}\}$ are approximately equal to
(a) $x^{(2)} = 1.0025\,, y^{(2)} = -0.9998, z^{(2)} = 0.9998$
(b) $x^{(2)} = -1.02\,, y^{(2)} = 0.965, z^{(2)} = 1.1515$
(c) $x^{(2)} = 1.02\,, y^{(2)} = 0.965, z^{(2)} = -1.1515$
(d) None of these</td>
<td></td>
</tr>
<tr>
<td>(x)</td>
<td>The value of the integral $\int_{0.2}^{1.4} (\sin x - log x + e^{x})\ dx$ estimated by using Simpson's 3/8th rule with 6 sub intervals is approximately equal to
(a) 12.0621
(b) 8.9983
(c) 4.053
(d) None of these</td>
<td>3M
CO3</td>
</tr>
<tr>
<td>(y)</td>
<td>From the table, the approximate value of x for which y is maximum is (Hint: Consider the terms only up to $\Delta^2 y_0$)
<table>
<tr><td>x</td><td>1.2</td><td>1.3</td><td>1.4</td><td>1.5</td><td>1.6</td></tr>
<tr><td>y</td><td>0.9320</td><td>0.9636</td><td>0.9855</td><td>0.9975</td><td>0.9996</td></tr>
</table>
(a) 0.56
(b) 1.576
(c) 2.143
(d) None of these</td>
<td>3M
CO3</td>
</tr>
</table>

Section – B (Attempt all the questions)
(5 × 8 marks)

2. The function $f(x) = e^{x} - 3x^{2}$ has two of its real roots near to 1.0 and 4.0. Find these two roots by fixed point iteration scheme by discussing whether these two roots could be obtained by same fixed point scheme or not.

[CO1,8 Marks]

3. The table below gives the velocity v of a body during the time t specified. Find its acceleration at $t = 1.15$ using an appropriate formula.

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

[CO2, 8 Marks]

4. Solve , by Crout's method, the following system of equations:
$x+y+z=3, 2x-y+3z=16, 3x+y-z=-3.$

[CO4,8 Marks]

5. Use fourth order Runge-Kutta method to solve the following simultaneous equations

$$\frac{dy}{dx}=-2y+4e^{-x}, \quad \frac{dz}{dx}=-\frac{yz^2}{3}$$

and obtain $y(0.2)$. Given $y(0)=2\ and\ z(0)=4$. Take $h=0.1$.

[CO5, 8 Marks]

6. Solve $\nabla^2 u=0$ in the square region bounded by $x=0, x=4, y=0, y=4$ and with boundary conditions $u(0,y)=0, u(4,y)=8+2y, u(x,0)=\frac{1}{2}x^2\ and\ u(x,4)=x^2$ taking *h*=*k*=*1*. (Perform 2 two iterations of Liebmann's process after obtaining the initial approximations using standard five point or diagonal five point formulae).

[CO5, 8 Marks]

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