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UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

Examination, July 2020

Programme: B.Tech. APE UP | **Semester : IV**

Course Name: Optimization techniques and Numerical Methods | **Max. Marks : 100**

Course Code: MATH 2013 | **Attempt Duration : 3 Hrs.**

No. of page/s: 13

Note:

1. Read the instruction carefully before attempting.
2. This question paper has two section, Section A and Section B.
3. There are total of seven questions in this question paper. **One** in **Section A** and **six** in **Section B**
4. **Section A** consist of multiple choice based questions and has the total weightage of 60%.
5. **Section B** consist of long answer based questions and has the total weightage of 40%.
6. The maximum time allocated to **Section A** is 120 minutes.
7. **Section B** to be submitted within 24 hrs from the scheduled time i.e. if the examination starts at 10:00 AM, the long answers must be submitted by 09:59:59 AM next day. Similarly, if the examination starts at 2:00 PM it must be submitted by 01:59:59 PM next day. (*Exceptional provision due extraordinary circumstance due to COVID-19 and due to internet connectivity issues in the far-flung areas*).
8. No submission of **Section B** shall be entertained after 24 Hrs.
9. **Section B** should be attempted after **Section A**
10. **The section B** should be attempted in blank white sheets (hand written) with all the details like programme, semester, course name, course code, name of the student, Sapid at the top (as in the format) and signature at the bottom (right hand side bottom corner)
11. Both section A & B should have questions from entire syllabus.
12. The COs mapping, internal choices within a section is same as earlier

Section – A (Attempt all the questions)
(60 marks)

S. No.		Marks	CO
Q 1.1	In solving a linear system of algebraic equations the following coefficient matrix A is obtained $$A = \begin{bmatrix} -4 & 1 & 2 \\ -2 & 3 & 0 \\ -2 & -2 & 5 \end{bmatrix}.$$ Which statements from the following are true? (Select all that apply) A. A is diagonally dominant matrix B. A is singular matrix C. A is not a diagonally dominant matrix D. A is non singular matrix	**1**	**CO1**
Q1.2	Given $$10x_1 - 2x_2 - x_3 - x_4 = 3$$ $$-2x_1 + 10x_2 - x_3 - x_4 = 15$$ $$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$ $$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$ The approximate solution of the given system of linear equations obtained from 2 iteration of Gauss Seidel method starting from the initial solution $x_1 = x_2 = x_3 = x_4 = 0$ is nearly close to A. $x_1 = 0.9, x_2 = 2, x_3 = 3, x_4 = -0.02$ B. $x_1 = 2, x_2 = 0.9, x_3 = 3, x_4 = -0.02$ C. $x_1 = 0.9, x_2 = 3, x_3 = 2, x_4 = -0.02$ D. $x_1 = 3, x_2 = 2, x_3 = 0.9, x_4 = -0.02$	**2**	**CO1**
Q1.3	What are the methods to solve linear system of algebraic equations? (Select all that apply) A. Runge Kutta method B. Gauss Seidel method	**1**	**CO3**

	C. Euler method D. SOR method		
Q1.4	Which one is the solution of the following system of linear equations? $$5x_1 + x_2 + x_3 + x_4 = 4$$ $$x_1 + 7x_2 + x_3 + x_4 = 12$$ $$x_1 + x_2 + 6x_3 + x_4 = -5$$ $$x_1 + x_2 + x_3 + 4x_4 = -6$$ A. $x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4$ B. $x_1 = 4, x_2 = 3, x_3 = 2, x_4 = 1$ C. $x_1 = 1, x_2 = 2, x_3 = -1, x_4 = -2$ D. $x_1 = -2, x_2 = -1, x_3 = 2, x_4 = 1$	**2**	**CO1**
Q1.5	The following system of linear equations is solved by SOR method with $w = 1.5$ $$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$ If SOR method is carried out starting with $x^{(1)} = [1 \ \ 2 \ \ 1]^T$ then which one is close to approximated solution obtained from first iteration of SOR method. A. $x = -1.25, y = 0.589, z = 5.995$ B. $x = -1.25, y = 0.125, z = 4.375$ C. $x = -2.22, y = 2.01, z = 3.875$ D. $x = 3.31, y = 2.01, z = -2.736$	**2**	**CO1**
Q1.6	Which is the formula for Newton-Raphson method to solve equation $f(x) = 0$? A. $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$ B. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ C. $x_{n+1} = x_n + \frac{f'(x_n)}{f(x_n)}$ D. $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$	**1**	**CO1**

Q1.7	If $f(x) = 0$ is nonlinear equation then which are correct statements about Newton-Raphson method? (Select all that apply) A. This method is useful in case of large values of $f'(x)$ B. If $f'(x) = 0$, the method fails C. This method converges provided the initial approximation x_0 is chosen sufficiently close to the root D. If $f'(x)$ is very large, the method fails	**1**	**CO1**
Q1.8	The graph of $y = 2\sin x$ and $y = \log_e x$ touch each other in the neighborhood of point $x = 8$. What is the coordinate of point of contact approximately? A. (8,5) B. (8,4) C. (8,3) D. (8,2)	**2**	**CO1**
Q1.9	Consider four points $x^{(1)} = (1,5)^T$, $x^{(2)} = (0,0)^T$, $x^{(3)} = (3,2)^T$ and $x^{(4)} = (3.396,0)^T$ to investigate Kunh-Tucher point (K-T point) of the following minimization problem $\min f(x) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$ subject to $(x_1 - 5)^2 + x_2^2 \leq 26$ $4x_1 + x_2 \leq 20$ $x_1, x_2 \geq 0$ Select all, which are correct. (Select all that apply) A. $x^{(1)}$ is K-T Point B. $x^{(2)}$ is K-T Point C. $x^{(3)}$ is K-T Point D. $x^{(4)}$ is K-T Point	**2**	**CO5**
Q1.10	What is the minimum number of iterations of the Newton-Raphson method required to find the root of the equation $e^{-x} = \sin x$ correct upto to 3 decimal places starting with $0 = 0.6$?	**2**	**CO1**

	A. 2 B. 3 C. 4 D. 5		
Q1.11	Which are Region-elimination techniques? (Select all that apply) A. exhaustive search B. interval halving C. golden section search D. steepest descent	**1**	**CO5**
Q1.12	What is the optimal solution of the following minimization problem using three iterations of interval halving method in the interval of uncertainty $(0,1)$? $\min f(x) = x(x - 1.5)$ A. 0.85 B. 0.75 C. 0.65 D. 0.55	**1**	**CO5**
Q1.13	Which methods are not used for interpolation? (Select all that apply) A. Lagrange method B. Cubic spline method C. Gauss Jacobi method D. Gauss elimination method	**1**	**CO5**
Q1.14	Which of the following methods are used for interpolation of unequally spaced data? (Select all that apply) A. Newton Gregory forward interpolation formula B. Lagrange interpolation formula C. Newton divided difference interpolation formula D. Newton Gregory backward interpolation formula	**1**	**CO2**
Q1.15	What is the value of $\Delta^{10}(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)$, when the interval of differencing being unity? A. $(10!)a$	**1**	**CO2**

	B. $(10!)b$ C. $(10!)c$ D. $(10!)abcd$		
Q.1.16	Values of x and y are given in the following table x: 2, 6, 10, 14, 18 y: -10, 8, 14, -19, 7 If $x_0 = 2$, $y_0 = -10$ and Δ is forward difference operator then what is the value of $\Delta^4 y_0$? A. 95 B. 125 C. 140 D. 155	**2**	**CO2**
Q1.17	Values of x and y are given in the following table x: 4, 6, 8, 10, 12 y: 5, -8, 10, 6, 2 What is the approximate value of $y(5)$? A. 10 B. -12 C. -6 D. -9	**2**	**CO2**
Q1.18	The following table gives the viscosity of an oil as a function of temperature. What is the approximate viscosity of oil at a temperature of 140^0 using Lagrange' s formula? $Temp^0$: 110, 130, 160, 190 $Viscosity$: 10.8, 8.1, 5.5, 4.8 A. 7 B. 8 C. 6	**1**	**CO2**

	D. 5		
Q1.19	Given: | x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | $f(x)$ | 1 | 0.5 | 0.2 | 0.1 | 0.0588235 | 0.0384615 | 0.0270270 | What is the approximated value of $\int_0^6 f(x)dx$ using Simpson's 1/3 rule? A. 1.9 B. 3.5 C. 1.3 D. 0.5	**1**	**CO3**
Q1.20	The table below shows the temperature $\theta(t)$ as a function of time t: | t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | $\theta(t)$ | 81 | 75 | 80 | 83 | 78 | 70 | 60 | What is the approximated value of $\int_1^7 \theta(t)dt$ using Simpson's 1/3 rule? A. 404 B. 440 C. 459 D. 505	**1**	**CO3**
Q1.21	What is the approximated value of $y'(1)$ from the following table? | x | 1 | 2 | 3 | 4 | 5 | 6 | | $y(x)$ | 198669 | 295520 | 389418 | 479425 | 564642 | 644217 | A. 55009 B. 98007 C. 70679 D. 56004	**2**	**CO3**

Q1.22 What is the approximated value of $f''(1.5)$ from the following table?

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.000	13.625	24.000	38.875	59.000

A. 9
B. 14
C. 18
D. 20

2 **CO3**

Q1.23 What is the approximated value of first derivative of $f(x)$ at $x = 0.4$ from the following table?

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

A. 0.5
B. 1.5
C. 2.5
D. 3

2 **CO3**

Q1.24 What is the approximated value of $\int_4^{5.2} y(x)dx$ using Simpson's $3/8$ rule from the following table?

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$y(x)$	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

A. 3.8278
B. 2.8278
C. 1.8278
D. 0.5678

2 **CO3**

Q1.25 Given that $\frac{dy}{dx} = \log_e(x + y)$, with the initial condition that $y = 1$ when $x = 0$. What is the approximated solution at $x = 0.5$ using Euler's method when step size $h = 0.1$?

A. 1
B. 2

2 **CO4**

	C. 3 D. 4		
Q1.26	Given that $\frac{dy}{dx} = xy^{\frac{1}{3}}$, with the initial condition that $y = 1$ when $x = 1$. What is the approximated solution at $x = 1.1$ using Runge-Kutta method when step size $h = 0.1$? A. 1 B. 2 C. 3 D. 4	**2**	**CO4**
Q1.27	In which part of the xy plane, the following equation is elliptic? $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + (x^2 + 4y^2)\frac{\partial^2 u}{\partial y^2} = 2\sin(xy)$$ A. In first quadrant B. Inside the circle $x^2 + y^2 = 2$ C. In the right side of $x = 2$ D. Outside the ellipse $\left(\frac{x}{0.5}\right)^2 + \left(\frac{y}{0.25}\right)^2 = 1$	**2**	**CO4**
Q1.28	Standard 5-point formula is used to solve A. Polynomial equation B. Laplace equation C. Algebraic equation D. Transcendental equation	**1**	**CO4**
Q1.29	Given: $\frac{dy}{dx} = e^x - y^2$ with $y(0) = 1$. What is the approximate value of y when $x = 0.2$ correct upto 3 decimal places using Taylor series method? A. 2.519 B. 1.019 C. 3.005 D. 4.555	**2**	**CO4**
Q1.30	The following boundary value problem is solved using finite difference method by taking number of subinterval $n = 4$	**2**	**CO4**

	$$x\frac{d^2y}{dx^2} + y = 0; \quad y(1) = 1, \qquad y(2) = 2$$ What are the values of $y(1.25)$, $y(1.5)$ and $y(1.75)$? A. $y(1.25) = 1.3513$, $y(1.5) = 1.6350$ and $y(1.75) = 1.8505$ B. $y(1.25) = 2.3513$, $y(1.5) = 2.6350$ and $y(1.75) = 2.8505$ C. $y(1.25) = 3.3513$, $y(1.5) = 3.6350$ and $y(1.75) = 3.8505$ D. $y(1.25) = 4.3513$, $y(1.5) = 4.6350$ and $y(1.75) = 4.8505$		
Q1.31	Given: $$2\frac{dy}{dx} = (1 + x^2)y^2 \text{ and } \quad y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21, \ y(0.4) = 1.28.$$ What is the approximated value of $y(0.5)$ using Milne's predictor corrector method? A. 1.4 B. 1.5 C. 1.6 D. 1.7	**2**	**CO4**
Q1.32	Which one is not a method to solve ODE initial value problem? A. Euler method B. Runge kutta method C. Taylor series method D. Secant method	**1**	**CO4**
Q1.33	Finite difference method is used to A. find definite integral B. evaluate maxima of function C. solve ODE-BVP D. transcendental equation	**1**	**CO4**
Q1.34	What are the methods to solve ODE? (Select all that apply) A. Finite difference method B. Taylor series method	**1**	**CO4**

	C. Runge kutta method D. Euler method		
Q1.35	What are the techniques to solve ODE boundary value problem? (Select all that apply) A. Euler method B. Newton Raphson method C. Shooting method D. Finite difference method	**1**	**CO4**
Q1.36	Which technique is used to solve Laplace equation? A. Euler method B. Finite difference method C. Newton Raphson method D. Shooting method	**1**	**CO4**
Q1.37	Which are the multi-objective optimization techniques? (Select all that apply) A. particle swarm B. genetic algorithm C. simulated annealing D. synthetic division	**1**	**CO6**
Q1.38	Jumping gene adaptation is used in A. interpolation B. integration C. differentiation D. optimization	**1**	**CO6**
Q1.39	Consider the following maximization problem $$\max f(x) = (3x_1^2 + 5x_2)^3 + x_1x_2$$ subject to $$3 \leq x_1 \leq 8, \quad -2 \leq x_2 \leq 5$$	**2**	**CO6**

<table>
<tr><td></td><td>In the following chromosome of length 10, first five positions represent x_1 and next five positions represent x_2

| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |

What are the real values of x_1 and x_2?

A. $x_1 = 7.1345, x_2 = 1.6864$
B. $x_1 = 7.3335, x_2 = 1.5864$
C. $x_1 = 7.4385, x_2 = 1.4864$
D. $x_1 = 7.5161, x_2 = 1.1613$</td><td></td><td></td></tr>
<tr><td>Q1.40</td><td>The fitness functions $minF1$ and $minF2$ for a multi-objective optimization problem corresponding to 9 chromosomes are given in the below table as:
<table><tr><td>Chromosome No.</td><td>C_1</td><td>C_2</td><td>C_3</td><td>C_4</td><td>C_5</td><td>C_6</td><td>C_7</td><td>C_8</td><td>C_9</td></tr><tr><td>Min F_1</td><td>2</td><td>3</td><td>3</td><td>4</td><td>4</td><td>5</td><td>5</td><td>5</td><td>6</td></tr><tr><td>Min F_2</td><td>7.5</td><td>6</td><td>7.5</td><td>5</td><td>6.5</td><td>4.5</td><td>6</td><td>7</td><td>6.5</td></tr></table>What are chromosomes at ranked 1 front? (Select all that apply)

A. C_1
B. C_2
C. C_4
D. C_6</td><td>2</td><td>CO6</td></tr>
</table>

Section – B (Attempt all the questions)
(40 marks)

<table>
<tr><td>Q2</td><td>Solve
$$\begin{bmatrix}2 & 1 & 0\\1 & 2 & 1\\0 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1\\2\\4\end{bmatrix}$$
Using SOR method with over-relaxation parameter $w = 1.5$. Carry out one iteration, starting with $x^{(1)} = [1 \;\; 2 \;\; 1]^T$.</td><td>5</td><td>CO1</td></tr>
<tr><td>Q3</td><td>From the following table, estimate the number of students who obtained 45 marks:
<table><tr><td>Marks (x)</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td></tr><tr><td>No. of students (y)</td><td>31</td><td>73</td><td>124</td><td>159</td><td>190</td></tr></table></td><td>5</td><td>CO2</td></tr>
<tr><td>Q4</td><td>The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data:
<table><tr><td>Time t (sec):</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td></tr><tr><td>Velocity v (m/sec):</td><td>0</td><td>3</td><td>14</td><td>69</td><td>228</td></tr></table></td><td>5</td><td>CO3</td></tr>
</table>

Q5	Using the finite difference method, find $y(0.25), y(0.5)$ and $y(0.75)$ satisfying the differential equation $\frac{d^2y}{dx^2} + y = x$, subject to the boundary conditions $y(0) = 0$, $y(1) = 2$.	**5**	**CO4**
Q6	Minimize the function, $f(x) = (3 - x_1^2)^2 + (2 - x_2^2)^2$ with the equality constraint $x_1 + x_2 = 2$ and inequality constraint $x_1 \geq 1$, using Kuhn-Tucker multiplier method.	**10**	**CO5**
Q7	The fitness functions $\min F_1$ and $\min F_2$ for a multi-objective optimization problem corresponding to 10 chromosomes are given in the below table as: (see table below) Draw the pareto optimal fronts and assign rank to the fronts. Also write (calculate) the crowding distance of each chromosome at highest rank fronts.	**10**	**CO6**

Chromosome No.	1	2	3	4	5	6	7	8	9	10
$Min\ F_1$	12	12	13	18	14	10	19	19	16	14
$Min\ F_2$	20	19	17	13	17	20	11	13	14	18

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