MODELING OF WATER FLOODING IN A RESERVOIR

(Final Project Report)

Submitted for Partial Fulfillment of the

Bachelor of Technology

(APPLIED PETROLEUM ENGINEERING)

Submitted To:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

DEHRADUN

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CERTIFICATE

This is to certify that the project work on *"MODELING OF WATER FLOODING IN A RESERVOIR*" submitted to University of Petroleum & Energy Studies, Dehradun, by Mr. Manvendra Singh Arya and Mr. Rohit Mitra, in partial fulfillment of the requirement for the award of Degree of Bachelor of Technology in Applied Petroleum Engineering (Academic Session 2003 – 2007) is a bonafide work carried out by them under my supervision and guidance. This work has not been submitted anywhere else for any other degree or diploma.

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Acknowledgement

With great pleasure we would like to express our sincere thanks to ***University of Petroleum & Energy Studies, Dehradun*** for giving us opportunity to carry out our final project work on the project ***"Modeling Of Water Flooding in Reservoir"***, in the due course of our B-Tech Program.

We are also very grateful to ***Dr. B.P. Pandey, Dean, College of Engineering, University of Petroleum & Energy Studies, Dehradun*** who guided us at each step during our project work. His much needed guidance and support provided us a solid platform for undergoing the process of learning during our project work.

We are deeply indebted to ***Dr. Sant Kumar (Reservoir Engineer's Consultant, Visiting Professor UPES)*** for his invaluable and erudite guidance, keen interest and constructive suggestions, timely and generous help beyond measure at all the stages during the progress of the project work.

Our sincere gratitude to ***Mr. Surendra Sahai (Professor, UPES)*** for his generous support in providing timely required help at various steps of completion of project.

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Abstract

Due to economics of a production system, the flow capacity of a well is a very important and therefore crucial to assess. It should be noted however that the flow capacity evolves over time and it tends to decrease as more fluid is produced.

To stabilize the current working production rates various methods are used as part of enhanced oil recovery methods. One of them being Water flooding of a reservoir, which is most common, economical, oldest and one of the efficient methods for increasing the depleting oil production rates within a reservoir.

This methods accounts for injecting pre-calculated amount of water within a reservoir from series of injection wells(with different injection configurations as patterns) to as to create a water front which pushes the oil front or bank from reservoir to production well. This can be done two ways

- ✓ Keeping production rate constant and injecting water accordingly
- ✓ Increasing the production rate by injecting more water so as reservoir pressure increases

There are methods for predicting the water flooding performances in a reservoir one of them being *Buckley and Leverett* One dimensional Flow. This gives the time at which the water will break out with oil at production well and thus efforts are made later to increase this break through time gap for better oil production. Various injection well patterns govern the recovery and Efficiency of water flooding thus this too needs to be considered

In this project we are trying to study the water flooding predictions in a reservoir by creating a computer model which is based on fractional flow equations and *Buckley and Leverett* One dimensional flow within a reservoir thus giving the break through saturation of the reservoir, also we try to calculate the percentage recovery of the reservoir under 5 spot pattern of injection wells.

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WATER FLOODING OF RESERVOIR

1.0 Introduction

The terms primary oil recovery, secondary oil recovery, and tertiary (enhanced) oil recovery are traditionally used to describe hydrocarbons recovered according to the method of production or the time at which they are obtained.

Primary oil recovery describes the production of hydrocarbons under the natural driving mechanisms present in the reservoir without supplementary help from injected fluids such as gas or water. In most cases, the natural driving mechanism is a relatively inefficient process and results in a low overall oil recovery. The lack of sufficient natural drive in most reservoirs has led to the practice of supplementing the natural reservoir energy by introducing some form of artificial drive, the most basic method being the injection of gas or water. Secondary oil recovery refers to the additional recovery that results from the conventional methods of water injection and immiscible gas injection.

Usually, the selected secondary recovery process follows the primary recovery but it can also be conducted concurrently with the primary recovery. Water flooding is perhaps the most common method of secondary recovery.

Tertiary (enhanced) oil recovery is that additional recovery over and above what could be recovered by primary and secondary recovery methods. Various methods of enhanced oil recovery (EOR) are essentially designed to recover oil, commonly described as residual oil, left in the reservoir after both primary and secondary recovery methods have been exploited to their respective economic limits

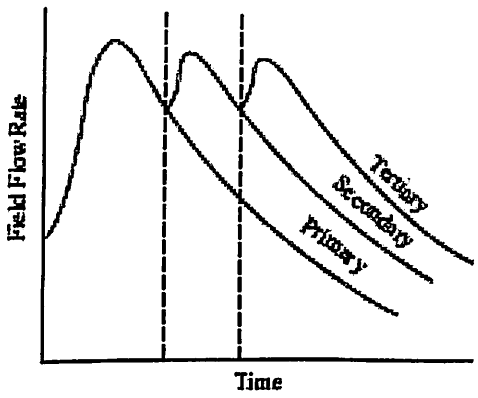


Fig 1.1 a

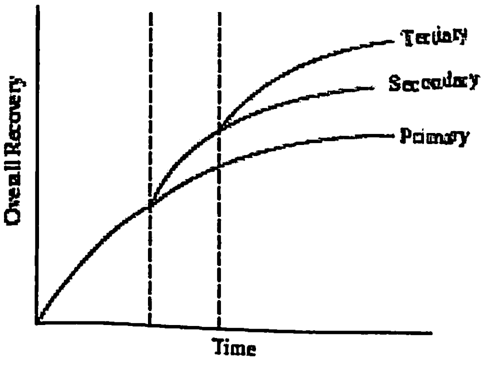


Fig 1.1 b

1.1 FACTORS TO CONSIDER IN WATERFLOODING

Thomas, Mahoney, and winter[12] (1989) pointed out that in determining the suitability of a candidate reservoir for water flooding, the following reservoir characteristics must be considered:

- ✓ Reservoir geometry
- ✓ Fluid properties
- ✓ Reservoir depth
- ✓ Lithology and rock properties
- ✓ Fluid saturations
- ✓ Reservoir uniformity and pay continuity
- ✓ Primary reservoir driving mechanism

1.2 OPTIMUM TIME TO WATERFLOOD

The most common procedure for determining the optimum time to start water flooding is to calculate:

- ✓ Anticipated oil recovery
- ✓ Fluid production rates
- ✓ Monetary investment
- ✓ Availability and quality of the water supply
- ✓ Costs of water treatment and pumping equipment
- ✓ Costs of maintenance and operation of the water installation facilities
- ✓ Costs of drilling new injection wells or converting existing production wells into injectors

These calculations should be performed for several assumed times and the net income for each case determined. The scenario that maximizes the profit and perhaps meets the operator's desirable goal is selected.

Cole[2] (1969) list the following factors as being important when determining the reservoir pressure (or time) to initiate a secondary recovery project:

1.2.1 Reservoir oil viscosity.

Water injection should be initiated when the reservoir pressure reaches its bubble-point pressure since the oil viscosity reaches its minimum value at this pressure. The mobility of the oil will increase with decreasing oil viscosity, which in turns improves the sweeping efficiency.

1.2.2 Free gas saturation.

(1) In **water injection projects**. It is desirable to have initial gas saturation, possibly as much as 10%. This will occur at a pressure that is below the bubble point pressure.
(2) In **gas injection projects.** Zero gas saturation in the oil zone is desired. This occurs while reservoir pressure is at or above bubble-point pressure.

1.2.3 Cost of injection equipment.

This is related to reservoir pressure, and at higher pressures, the cost of injection equipment increases. Therefore, a low reservoir pressure at initiation of injection is desirable.

1.2.4 Productivity of producing wells.

A high reservoir pressure is desirable to increase the productivity of producing wells, which prolongs the flowing period of the wells, decreases lifting costs, and may shorten the overall life of the project.

1.2.5 Effect of delaying investment on the time value of money.

A delayed investment in injection facilities is desirable from this standpoint.

1.2.6 Overall life of the reservoir.

Because operating expenses are an important part of total costs, the fluid injection process should be started as early as possible. Some of these six factors act in opposition to others. Thus the actual pressure at which a fluid injection project should be initiated will require optimization of the various factors in order to develop the most favorable overall economics.

1.3 SELECTION OF FLOODING PATTERNS

One of the first steps in designing a waterflooding project is flood pattern selection. The objective is to select the proper pattern that will provide the injection fluid with the maximum possible contact with the crude oil system. This selection can be achieved by

(1) Converting existing production wells into injectors or

(2) Drilling infill injection wells. When making the selection, the following factors must be considered:

- ✓ Reservoir heterogeneity and directional permeability
- ✓ Direction of formation fractures
- ✓ Availability of the injection fluid (gas or water)
- ✓ Desired and anticipated flood life
- ✓ Maximum oil recovery
- ✓ Well spacing, productivity, and injectivity

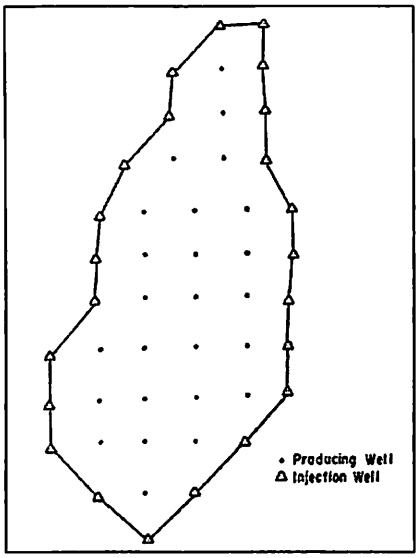
In general, the selection of a suitable flooding pattern for the reservoir depends on the number and location of existing wells. In some cases, producing wells can be converted to injection wells while in other cases it may be necessary or desirable to drill new injection wells.

Essentially four types of well arrangements are used in fluid injection projects:

- ✓ Irregular injection patterns
- ✓ Peripheral injection patterns
- ✓ Regular injection patterns
- ✓ Crestal and basal injection patterns

1.3.1 Irregular Injection Patterns

Willhite[11] (1986) points out that surface or subsurface topology and/or the use of slant-hole drilling techniques may result in production or injection wells that are not uniformly located. In these situations, the region affected by the injection well could be different for every injection well. Some small reservoirs are developed for primary production with a limited number of wells and when the economics are marginal, perhaps only few production wells are converted into injectors in a non-uniform pattern. Faulting and localized variations in porosity or permeability may also lead to irregular patterns.



1.3.2 Peripheral Injection Patterns

In peripheral flooding, the injection wells are located at the external boundary of the reservoir and the oil is displaced toward the interior of the reservoir, as shown in Figure Craig ^{4}(1971), in an excellent review of the peripheral flood, points out the following main characteristics of the flood

- ✓ The peripheral flood generally yields a maximum oil recovery with a minimum of produced water.
- ✓ The production of significant quantities of water can be delayed until only the last row of producer remains.
- ✓ Because of the unusually small number of injectors compared with the number of producers, it takes a long time for the injected water to fill up the reservoir gas space. The result is a delay in the field response to the flood.
- ✓ For a successful peripheral flood, the formation permeability must be large enough to permit the movement of the injected water at the desired rate over the distance of several well spacing from injection wells to the last line of producers.
- ✓ To keep injection wells as close as possible to the waterflood front without bypassing any movable oil, watered-out producers may be converted into injectors. However, moving the location of injection wells frequently requires laying longer surface water lines and adding costs.

- ✓ Results from peripheral flooding are more difficult to predict. The displacing fluid tends to displace the oil bank past the inside producers, which are thus difficult to produce.
- ✓ Injection rates are generally a problem because the injection wells continue to push the water greater distances.

1.3.3 Regular Injection Patterns

Due to the fact that oil leases are divided into square miles and quarter square miles, fields are developed in a very regular pattern. A wide variety of injection-production well arrangements have been used in injection projects. The most common patterns, as shown in Figure 1.3.1 are the following:

1.3.3.1 Direct line drive.

The lines of injection and production are directly opposed to each other. The pattern is characterized by two parameters:

a = distance between wells of the same type, and d = distance between lines of injectors and producers.

1.3.3.2 Staggered line drive. The wells are in lines as in the direct line, but the injectors and producers are no longer directly opposed but laterally displaced by a distance of a/2.

1.3.3.3 Five spot. This is a special case of the staggered line drive in which the distance between all like wells is constant, i.e., a = 2d. Any four injection wells thus form a square with a production well at the center.

1.3.3.4 Seven spot. The injection wells are located at the corner of a hexagon with a production well at its center.

1.3.3.5 Nine spot. This pattern is similar to that of the five spot but with an extra injection well drilled at the middle of each side of the square. The pattern essentially contains eight injectors surrounding one producer.

The patterns termed **inverted** have only one injection well per pattern. This is the difference between **normal** and **inverted** well arrangements. Note that the four-spot and inverted seven-spot patterns are identical.

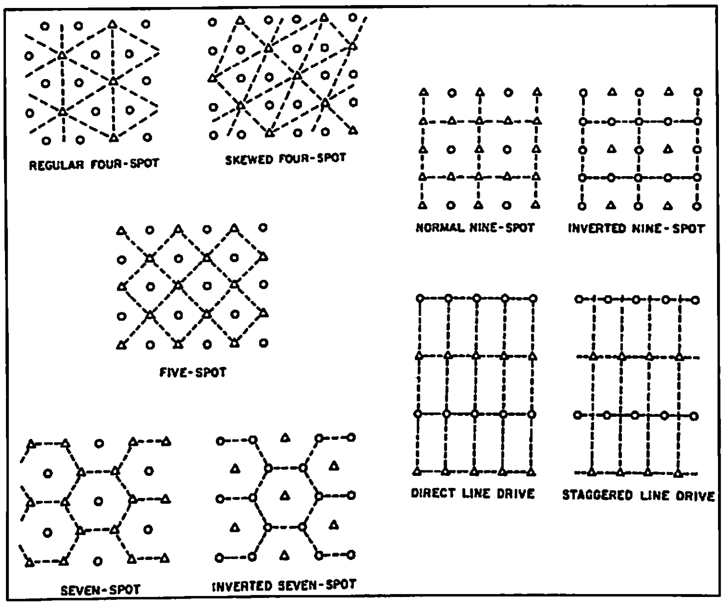


Fig 1.3.1

1.3.4 Crestal and Basal Injection Patterns

In crestal injection, as the name implies, the injection is through wells located at the top of the structure. Gas injection projects typically use a crestal injection pattern. In basal injection, the fluid is injected at the bottom of the structure. Many water-injection projects use basal injection patterns with additional benefits being gained from gravity segregation. A schematic illustration of the two patterns is shown in Figure 1.3.4

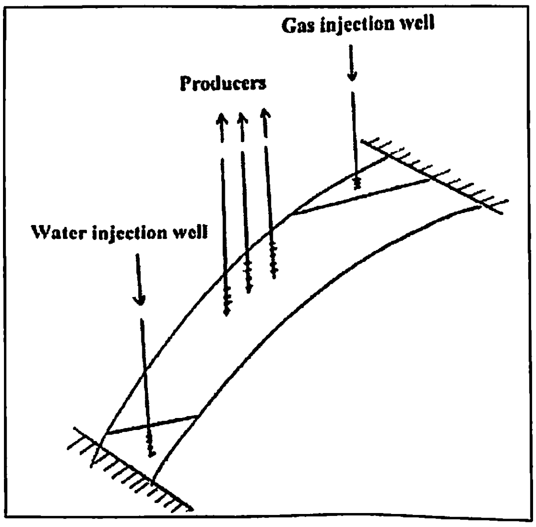


Fig 1.3.4

2.0 OVERALL RECOVERY EFFICIENCY

The overall recovery factor (efficiency) RF of any secondary or tertiary oil recovery method is the product of a combination of three individual efficiency factors as given by the following generalized expression:

$$RF = (E_D)(E_A)(E_V)$$

In terms of cumulative oil production, Equation can be written as:

$$N_P = N_S E_D E_A E_V$$

where RF = overall recovery factor

N_S = initial oil in place at the start of the flood, STB

N_P = cumulative oil produced, STB

E_D = displacement efficiency

E_A = areal sweep efficiency

E_V = vertical sweep efficiency

The displacement efficiency ED is the fraction of movable oil that has been displaced from the swept zone at any given time or pore volume injected. Because an immiscible gas injection or waterflood will always leave behind some residual oil, ED will always be less than 1.0.

The areal sweep efficiency EA is the fractional area of the pattern that is swept by the displacing fluid. The major factors determining areal sweep are:

- ✓ Fluid mobilities
- ✓ Pattern type
- ✓ Areal heterogeneity
- ✓ Total volume of fluid injected

The vertical sweep efficiency EV is the fraction of the vertical section of the pay zone that is contacted by injected fluids. The vertical sweep efficiency is primarily a function of:

- ✓ Vertical heterogeneity
- ✓ Degree of gravity segregation
- ✓ Fluid mobilities
- ✓ Total volume injection

The product of E_A E_V is called the **volumetric sweep efficiency** and represents the overall fraction of the flood pattern that is contacted by the injected fluid.

All three efficiency factors (i.e., E_D, E_A, and E_V) are variables that increase during the flood and reach maximum values at the economic limit of the injection project. Each of the three efficiency factors is discussed individually and methods of estimating these efficiencies are presented.

2.1 DISPLACEMENT EFFICIENCY

As defined previously, displacement efficiency is the fraction of movable oil that has been recovered from the swept zone at any given time.

Mathematically, the displacement efficiency is expressed as:

$$E_D = \frac{\text{Volume of oil at start of flood} - \text{Remaining oil volume}}{\text{Volume of oil at start of flood}}$$

$$E_D = \frac{(\text{Pore volume})\left(\frac{S_{oi}}{B_{oi}}\right) - (\text{Pore volume})\left(\frac{\overline{S}_o}{B_o}\right)}{(\text{Pore volume})\left(\frac{S_{oi}}{B_{oi}}\right)}$$

$$E_D = \frac{\frac{S_{oi}}{B_{oi}} - \frac{\overline{S}_o}{B_o}}{\frac{S_{oi}}{B_{oi}}} \quad ..(1)$$

where S_{oi} = initial oil saturation at start of flood
B_{oi} = oil FVF at start of flood, bbl/STB
$\overline{S}_o$ = average oil saturation in the flood pattern at a particular point during the flood

Assuming a constant oil formation volume factor during the flood life, above equation (1) is reduced to:

$$E_D = \frac{S_{oi} - \overline{S}_o}{S_{oi}}$$

.....(2)

where the initial oil saturation Soi is given by:

$$S_{oi} = 1 - S_{wi} - S_{gi}.$$

However, in the swept area, the gas saturation is considered zero, thus:

$$\overline{S}_o = 1 - \overline{S}_w.$$

The displacement efficiency ED can be expressed more conveniently in terms of water saturation by substituting the above relationships into Equation, to give:

$$E_D = \frac{\overline{S}_w - S_{wi} - S_{gi}}{1 - S_{wi} - S_{gi}} \qquad ......(3)$$

where $\overline{S}_w$ = average water saturation in the swept area
S_{gi} = initial gas saturation at the start of the flood
S_{wi} = initial water saturation at the start of the flood

If no initial gas is present at the start of the flood, above equation is reduced to:

$$E_D = \frac{\overline{S}_w - S_{wi}}{1 - S_{wi}} \qquad ....(4)$$

The displacement efficiency ED will continually increase at different stages of the flood, i.e., with increasing Sw. Equation (2) or (4) suggests that ED reaches its maximum when the average oil saturation in the area of the flood pattern is reduced to the residual oil saturation Sor or, equivalently.

2.2 AREAL SWEEP EFFICIENCY

The areal sweep efficiency EA is defined as the fraction of the total flood pattern that is contacted by the displacing fluid. It increases steadily with injection from zero at the start of the flood until breakthrough occurs, after which EA continues to increase at a slower rate.

The areal sweep efficiency depends basically on the following three main factors:

- ✓ Mobility ratio M
- ✓ Flood pattern
- ✓ Cumulative water injected W_{inj}

Mobility Ratio

In general, the mobility of any fluid λ is defined as the ratio of the effective permeability of the fluid to the fluid viscosity, i.e.:

$$\lambda_o = \frac{k_o}{\mu_o} = \frac{k\,k_{ro}}{\mu_o}$$

$$\lambda_w = \frac{k_w}{\mu_w} = \frac{k\,k_{rw}}{\mu_w}$$

$$\lambda_g = \frac{k_g}{\mu_g} = \frac{k\,k_{rg}}{\mu_g}$$

where λ_o, λ_w, λ_g = mobility of oil, water, and gas, respectively
k_o, k_w, k_g = effective permeability to oil, water, and gas, respectively
k_{ro}, k_{rw} = relative permeability to oil, water, and gas, respectively
k = absolute permeability

The fluid mobility as defined mathematically by the above three relationships indicates that λ is a strong function of the fluid saturation. The mobility ratio M is defined as the mobility of the *displacing fluid* to the mobility of the *displaced fluid*, or:

$$M = \frac{\lambda_{displacing}}{\lambda_{displaced}}$$

For waterflooding then:

$$M = \frac{\lambda_w}{\lambda_0}$$

Simplifying gives:

$$M = \frac{k_{rw}}{k_{ro}} \frac{\mu_o}{\mu_w} \quad ....(5)$$

Muskat[8] (1946) points out that in calculating M by applying Equation(5), the following concepts must be employed in determining kro and krw:

2.2.1 Relative permeability to oil kro. Because the displaced oil is moving ahead of the water front in the non invaded portion of the pattern, as shown schematically in Figure 2.2, kro must be evaluated at the initial water saturation Swi.

2.2.2 Relative permeability to water krw. The displacing water will form a water bank that is characterized by an average water saturation of S_{wBT} in the swept area. This average saturation will remain constant until breakthrough, after which the average water saturation will continue to increase (as denoted by Sw2). The mobility ratio, therefore, can be expressed more explicitly under two different stages of the flood:

From the start to breakthrough:

$$M = \frac{k_{rw} @ \overline{S}_{wBT}}{k_{ro} @ S_{wi}} \frac{\mu_0}{\mu_w}$$

where $k_{rw} @ \overline{S}_{wBT}$ = relative permeability of water at $\overline{S}_{wBT}$

k_{ro} @ S_{wi} = relative permeability of oil at S_{wi}

The above relationship indicates that the mobility ratio will remain constant from the start of the flood until breakthrough occurs.

After breakthrough:

$$M = \frac{k_{rw} @ \overline{S}_{w2}}{k_{ro} @ S_{wi}} \frac{\mu_0}{\mu_w}$$

Above Equation indicates that the mobility of the water krw/μw will increase after breakthrough due to the continuous increase in the average water saturation. This will result in a proportional increase in the mobility ratio M after breakthrough, as shown in Figure 2.2

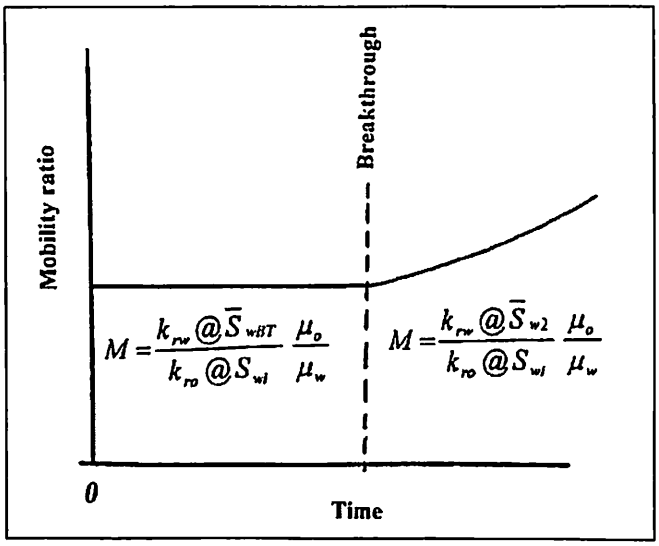


Fig 2.2

In general, if no further designation is applied, the term **mobility ratio** refers to the mobility ratio before breakthrough.

3.0 THE FRACTIONAL FLOW EQUATION

The development of the fractional flow equation is attributed to Leverett[1] (1941). In this, and the following two sections, oil displacement will be assumed to take place under the so-called diffuse flow condition. This means that fluid saturations at any point in the linear displacement path are uniformly distributed with respect to thickness. The sole reason for making this assumption is that it permits the displacement to be described, mathematically, in one dimension and this provides the simplest possible model of the displacement process. The one dimensional description follows from the fact that since the water saturation is uniformly distributed in the dip-normal direction then so too are the relative permeabilities to oil and water, which are themselves functions of the water saturation at any point. This means that the simultaneous flow of oil and water can be modeled using thickness averaged relative permeabilities, along the centre line of the reservoir, which are also equivalent to relative permeabilities at any point throughout the thickness.

The diffuse flow condition can be encountered under two extreme physical conditions:

a) when displacement occurs at very high injection rates so that, the condition of vertical equilibrium is not satisfied and the effects of the capillary and gravity forces are negligible, and

b) for displacement at low injection rates in reservoirs for which the measured capillary transition zone greatly exceeds the reservoir thickness (H >> h) and the vertical equilibrium condition applies.

The latter case can be visualized by considering the capillary pressure curve, fig 3.0 Since, H >> h then it will appear that the water saturation is, to a first approximation, uniformly distributed with respect to thickness in the reservoir

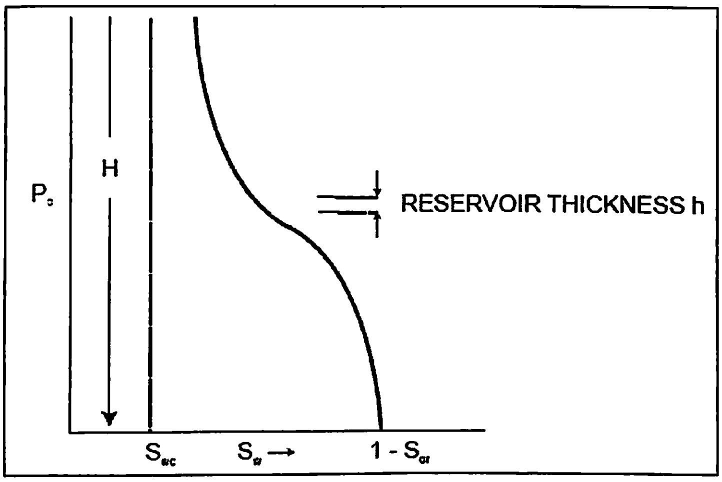


Fig 3.0

It should also be noted that relative permeabilities are measured in the laboratory under the diffuse flow condition. This normally results from displacing one fluid by another, in thin core plugs, at high flow rates. As such, the laboratory, or rock relative permeabilities, must be regarded as point relative permeabilities which are functions of the point water saturation in the reservoir. It is, therefore, only when describing displacement, under the diffuse flow condition, that rock relative permeabilities can be used directly in calculations since, in this case, they also represent the thickness averaged relative permeabilities.

Consider then, oil displacement in a tilted reservoir block, as shown in figure, which has a uniform cross sectional area A.

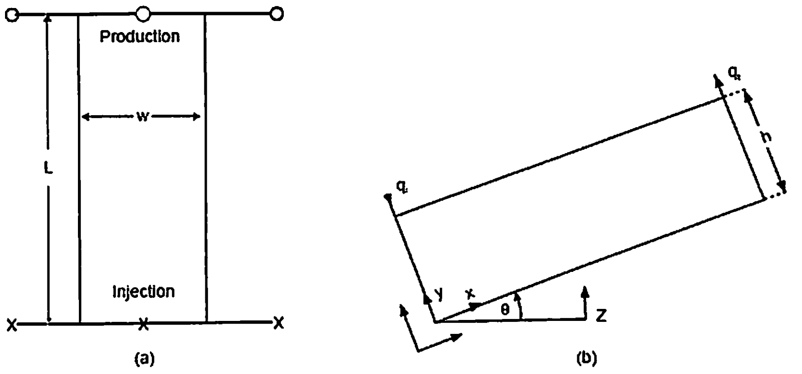


Fig Linear prototype reservoir model, (a) plan view; (b) cross section

Applying Darcy's law, for linear flow, the one dimensional equations for the simultaneous flow of oil and water are

$$q_o = -\frac{kk_{ro}A\rho_o}{\mu_o}\frac{\partial\Phi_o}{\partial x} = -\frac{kk_{ro}A}{\mu_o}\left(\frac{\partial p_o}{\partial x} + \frac{\rho_o g\ \sin\theta}{1.0133\times10^6}\right)$$

and

$$q_w = -\frac{kk_{rw}A\rho_w}{\mu_w}\frac{\partial\Phi_w}{\partial x} = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial p_w}{\partial x} + \frac{\rho_w g\ \sin\theta}{1.0133\times10^6}\right)$$

By expressing the oil rate as

qo = qt - qw

the subtraction of the above equations gives

$$q_w = -\left(\frac{\mu_{rw}}{kk_{rw}} + \frac{\mu_o}{kk_{ro}}\right) = \frac{q_t\mu_o}{kk_{ro}} + A\left(\frac{\partial P_c}{\partial x} - \frac{\Delta\rho\ g\sin\theta}{1.0133\times10^6}\right) \qquad \text{................(8)}$$

in which

$$\frac{\partial P_c}{\partial x} = \frac{\partial p_o}{\partial x} - \frac{\partial p_w}{\partial x}$$

the capillary pressure gradient in the direction of flow, and

$$\Delta\rho = \rho_W - \rho_O$$

The fractional flow of water, at any point in the reservoir, is defined as

$$f_w = \frac{q_w}{q_o + q_w} = \frac{q_w}{q_t}$$

and substitution of this in equation, (8) gives

$$f_w = \frac{1 + \frac{kk_{ro}A}{q_t\mu_o}\left(\frac{\partial P_c}{\partial x} - \frac{\Delta\rho\ g\ \sin\theta}{1.0133\times10^6}\right)}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \qquad \text{.........(9)}$$

this equation can be expressed in field units as

$$f_w = \frac{1 + 1.127\times10^{-3}\ \frac{kk_{ro}A}{q_t\mu_o}\left(\frac{\partial P_c}{\partial x} - .4335\ \Delta\gamma\sin\theta\right)}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \qquad \text{......................(10)}$$

both of these being fractional flow equations for the displacement of oil by water, in one dimension.

It is worthwhile considering the influence of the various component parts of this expression. According to the convention adopted in this text θ is the angle measured from the horizontal to the line indicating the direction of flow. Therefore, the gravity term $\Delta\rho g \sin\theta/1.0133*10^6$ will be positive for oil displacement in the up dip direction ($0 < \theta < \pi$), as shown in fig.), and negative for displacement down-dip ($\pi < \theta < 2\pi$).

As a result, provided all the other terms in equation (9) are the same, the fractional flow of water for displacement up-dip is lower than for displacement down-dip since in the former case gravity tends to suppress the flow of water.

For displacement in a horizontal reservoir ($\sin\theta = 0$), and neglecting, for the moment, the capillary pressure gradient, the fractional flow equation is reduced to

$$f_w = \frac{1}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \qquad \text{.........(11)}$$

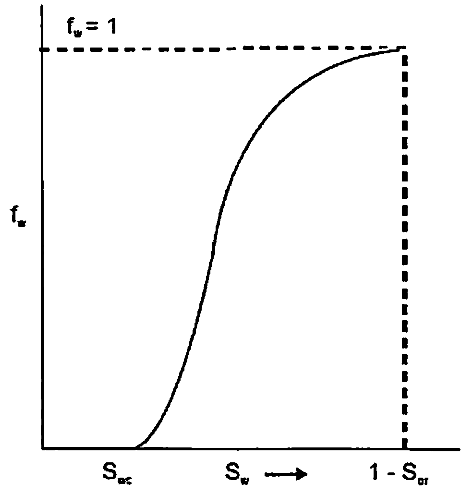


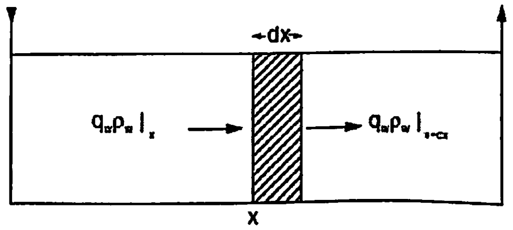
Fig. Typical fractional flow curve as a function of water saturation, equ. (11)

3.1 BUCKLEY-LEVERETT ONE DIMENSIONAL DISPLACEMENT

Buckley and Leverett[1] presented what is recognized as the basic equation for describing immiscible displacement in one dimension. For water displacing oil, the equation determines the velocity of a plane of constant water saturation traveling through a linear system. Assuming the diffuse flow condition, the conservation of mass of water flowing through volume element $A\varphi dx$, fig. may be expressed as

Mass flow rate In - Out = Rate of increase of mass in the volume element

$$q_w\rho_w\Big|_x - q_w\rho_w\Big|_{x+dx} = A\phi\ dx\frac{\partial}{\partial t}(\rho_w S_w) \qquad ....(12)$$



Mass flow rate of water through a linear volume element $A\phi dx$

$$q_w\rho_w\Big|_x - \left(q_w\rho_w\Big|_x + \frac{\partial}{\partial x}(q_w\rho_w)dx\right) = A\phi\ dx\frac{\partial}{\partial t}(\rho_w S_w)$$

which can be reduced to

$$\frac{\partial}{\partial x}(q_w\rho_w) = -A\phi\frac{\partial}{\partial t}(\rho_w S_w) \qquad ......(13)$$

and for the assumption of incompressible displacement

$$\left.\frac{\partial q_w}{\partial x}\right|_t = -A\phi\left.\frac{\partial S_w}{\partial t}\right|_x \qquad ....(14)$$

The full differential of the water saturation is

$$dS_w = \left.\frac{\partial S_w}{\partial x}\right|_t dx + \left.\frac{\partial S_w}{\partial t}\right|_x dt$$

and since it is the intention to study the movement of a plane of constant water saturation, that is, dSw = 0, then

$$\left.\frac{\partial S_w}{\partial t}\right|_x = -\left.\frac{\partial S_w}{\partial x}\right|_t \left.\frac{dx}{dt}\right|_{S_w} \qquad ....(15)$$

Furthermore

$$\left.\frac{\partial q_w}{\partial x}\right|_t = \left(\frac{\partial q_w}{\partial S_w}\cdot\frac{\partial S_w}{\partial x}\right)_t \qquad ....(16)$$

and substituting equations (15) and (16) in equation (14) gives

$$\left.\frac{\partial q_w}{\partial S_w}\right|_t = A\phi\left.\frac{dx}{dt}\right|_{S_w} \qquad .... (17)$$

Again, for incompressible displacement, qt is constant and, since qw = qtfw, equation (17) may be expressed as

$$v_{S_w} = \left.\frac{dx}{dt}\right|_{S_w} = \frac{q_t}{A\phi}\left.\frac{df_w}{dS_w}\right|_{S_w} \qquad ....(18)$$

This is the equation of Buckley-Leverett which implies that, for a constant rate of water injection (qt = qi), the velocity of a plane of constant water saturation is directly proportional to the derivative of the fractional flow equation evaluated for that saturation.

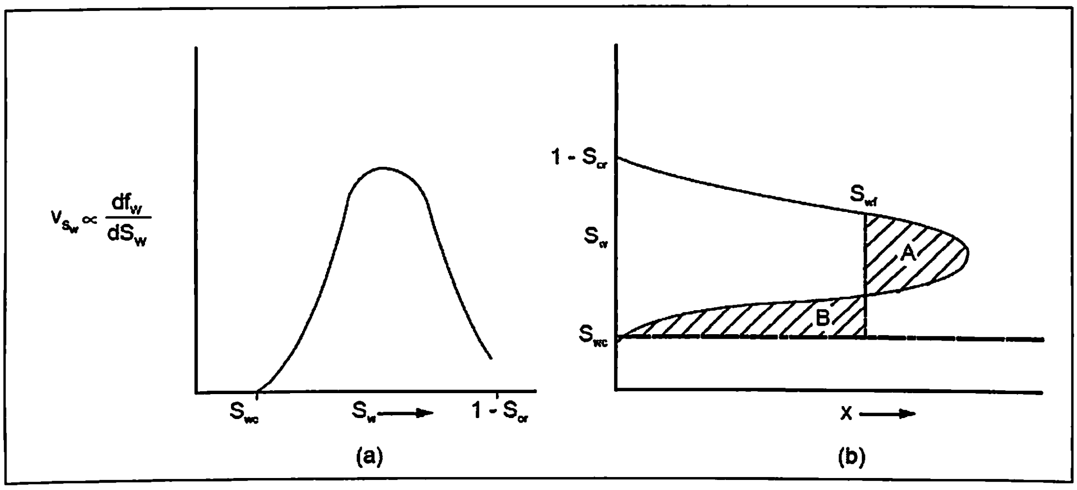
This is the equation of Buckley-Leverett[1] which implies that, for a constant rate of water injection (qt = qi), the velocity of a plane of constant water saturation is directly proportional to the derivative of the fractional flow equation evaluated for that saturation. If the capillary pressure gradient term is neglected in equ. (9) then the fractional flow is strictly a function of the water saturation, irrespective of whether the gravity term is included or not, hence the use of the total differential of fw in the Buckley-Leverett equation. Integrating for the total time since the start of injection gives

$$x_{S_w} = \frac{1}{A\phi}\frac{df_w}{dS_w}\int_0^t q_t dt$$

$$x_{S_w} = \frac{W_i}{A\phi}\left.\frac{df_w}{dS_w}\right|_{S_w} \qquad \text{...........(19)}$$

Where Wi is the cumulative water injected and it is assumed, as an initial condition, that Wi = 0 when t = 0. Therefore, at a given time after the start of injection (Wi = constant) the position of different water saturation planes can be plotted, using equ. (19), merely by determining the slope of the fractional flow curve for the particular value of each saturation.

There is a mathematical difficulty encountered in applying this technique which can be appreciated by considering the typical fractional flow curve shown in fig. 10.9 in conjunction with equ. (19). Since there is frequently a point of inflexion in the fractional flow curve then the plot of dfw/dSw versus Sw will have a maximum point, as shown in fig.(a) below. Using equation (19) to plot the saturation distribution at a particular time will therefore result in the solid line shown in fig.(b) below. This bulbous saturation profile is physically impossible since it indicates that multiple water saturations can co-exist at a given point in the reservoir. What actually occurs is that the intermediate values of the water saturation, which as shown in fig.(a) below have the maximum velocity, will initially tend to overtake the lower saturations resulting in the formation of a saturation discontinuity or shock front.



(a) Saturation derivative of a typical fractional flow curve and (b) resulting water saturation distribution in the displacement path

Because of this discontinuity the mathematical approach of Buckley-Leverett, which assumes that Sw is continuous and differentiable, will be inappropriate to describe the situation at the front itself. Behind the front, however, in the saturation range

$$S_{wf} < S_w < 1 - S_{or}$$

where Swf is the shock front saturation, equations (18) and (19) can be applied to determine the water saturation velocity and position. Furthermore, in this saturation range the capillary pressure gradient is usually negligible, as noted in the previous section, and the fractional flow equation to be used in equation s. (18) and (19) is simply

$$f_w = \frac{1}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \quad . \qquad \ldots\ldots\ldots\ldots (11)$$

in a horizontal reservoir, or

$$f_w = \frac{1 - \frac{k k_{ro} A}{q_t \mu_o} \frac{\Delta\rho \ g \ \sin\theta}{1.0133 \times 10^6}}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \qquad \ldots\ldots (9)$$

In a dipping reservoir. To draw the correct water saturation profile using the Buckley-Leverett technique requires the determination of the vertical dashed line, shown in fig. (b) Up, such that the shaded areas A and B are equal. The dashed line then represents the shock front saturation discontinuity.

3.3.1 Cumulative Water Injected

Continued injection after breakthrough can result in substantial increases in recovery, especially in the case of an adverse mobility ratio. The work of Craig et al. (1955) has shown that significant quantities of oil may be swept by water after breakthrough. It should be pointed out that the higher the mobility ratio, the more important is the "after-breakthrough" production.

4.0 WATERFLOOD DESIGN SCHEME

4.1 Introduction

The design of a waterflood involves both technical and economic considerations. Economic analyses are based on estimates of waterflood performance. These estimates maybe rough or sophisticated depending on the requirements of a particular project and the philosophy of the operator. This chapter presents methods of estimating waterflood performance for economic analyses. It is organized in order of increasing complexity beginning with first-pass estimates with simple methods and ending with an introduction to the capability of reservoir simulators to evaluate waterflood designs.

4.1.1 Factors Constituting a Design

The five steps in the design of a waterflood are as follows.

- ✓ Evaluation of the reservoir, including primary production performance.
- ✓ Selection of potential flooding plans.
- ✓ Estimation of injection and production rates.
- ✓ Projection of oil recovery over the anticipated life of the project for each flooding plan.
- ✓ Identification of variables that may cause uncertainty in the technical analysis.

Technical analysis of a waterflood produces estimates of the volumes of fluids and rates. Those estimates are used also for sizing equipment and fluid-handling systems. It is necessary to identify a source of water for injection that is compatible with connate fluids a swell as with reservoir rock. Design includes arrangements for proper disposal of produced water.

4.1.2 Reservoir description

The purposes of a reservoir description in waterflood design are

- ✓ To define the areal and vertical extent of the reservoir,
- ✓ To describe quantitatively the variation in rock properties- such as permeability and porosity within the reservoir,

- ✓ To determine the primary production mechanism, including estimates of the oil remaining to be produced under primary operation,
- ✓ To estimate the distribution of the oil resource in the reservoir, and
- ✓ To evaluate fluid properties required for predicting waterflood performance.
- ✓ The data and interpretations that are obtained in developing a reservoir description make up many of the input data for the waterflood design.

4.1.3 Reservoir Characteristics

- ✓ Areal and vertical extent of producing formation.
- ✓ Isopach maps of gross and net sand.
- ✓ Correlation of layers and other zones.

4.1.4 Reservoir Rock Properties

- ✓ Areal variation of average permeability, including directional trends derived from geological interpretations.
- ✓ Areal variation of porosity.
- ✓ Reservoir heterogeneity-particularly the variation of permeability with thickness and zone.

4.1.5 Reservoir Fluid Properties

- ✓ Gravity, FVF, and viscosity as a function of reservoir pressure.

4.1.6 Primary Producing Mechanism

- ✓ Identification of producing mechanisms-such as fluid expansion, solution-gas drive, or water drive.
- ✓ Existence of gas caps or aquifers.
- ✓ Estimation of oil remaining to be produced under primary operations.
- ✓ Pressure distribution in the reservoir.

4.1.7 Distribution of Oil Resources in Reservoir at Beginning of Waterflood

- ✓ Trapped-gas saturation from solution-gas drive.
- ✓ Vertical variation of saturation as a result of gravity segregation.

- ✓ Presence of mobile connate water.
- ✓ Areas already waterflooded by natural water drive

4.1.8 Rock/Fluid Properties

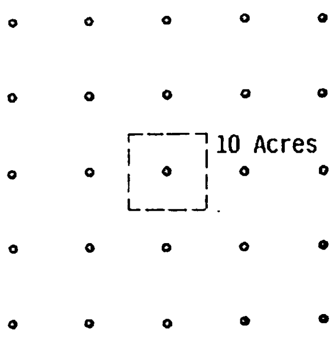
- ✓ Relative permeability data for the reservoir rock.
- ✓ Minimum data are endpoints of relative permeability

5.0 Selection of Potential Flooding Plans

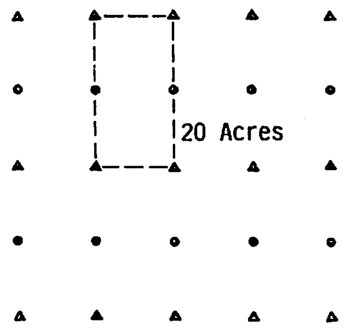
Selection of the waterflooding plan is determined by factors that are often unique to each reservoir. In some reservoirs, the waterflood may be done with edge wells to form a peripheral flood. This is called pressure maintenance when water injection supplements declining reservoir energy from solution-gas drive or an aquifer of limited extent. Pressure maintenance often begins while the reservoir is still under primary operation to maintain maximum production rates. Pattern flooding, an alternative to pressure maintenance, may be selected because reservoir properties will not permit waterflooding through edge wells at desired injection rates. In pattern flooding, injection and withdrawal rates are determined by well spacing as well as reservoir properties. Pattern size becomes a variable that is considered in economic analyses.

The selection of possible waterflooding patterns depends on existing wells that generally must be used because of economics. Pattern selection is constrained by the locations of production wells. Many fields are developed for primary production on a uniform well spacing as in Fig. If only existing wells are used, the options are limited to a line drive, five-spot, or nine-spot pattern as shown in Figs. 6.1B through 6.ID. Infill drilling of injection wells can be used to reduce the spacing, as in Fig. 6.2. Any of these patterns can be used to waterflood a reservoir, but final selection of spacing and pattern type, when there are several possibilities, is determined by comparison of the economics of alternative flooding schemes. Surface or subsurface topology and/or the use of slant hole drilling techniques may result in production or injection wells that are non-uniformly located, as shown in Fig. below.

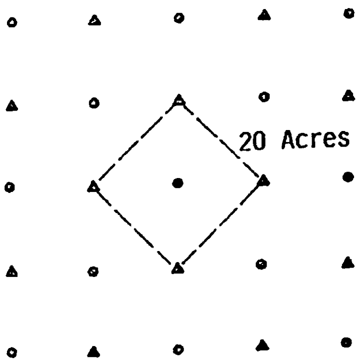
In these situations, the flooding pattern- i.e., the region affected by the injection well- could be different for every injection well. Some reservoirs are small and are developed for primary production with a limited number of wells. When economics are marginal, one producer may be converted to an injection well,



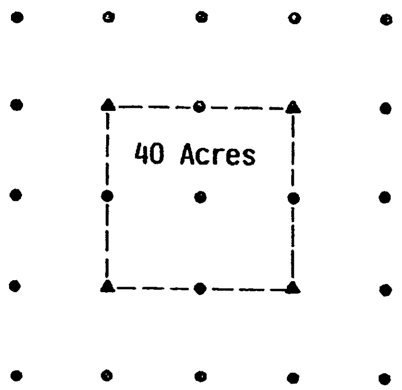
Regular well spacing during primary production period-10-acre spacing.



Line drive spacing on 20-acre spacing.



Five-spot pattern on 20-acre spacing.



Nine-spot pattern on 40-acre spacing.

Reservoir characteristics-such as gas cap, water drive, or a fault-may limit the options that can be considered. Edge or peripheral drives may be more appropriate than the pattern flood, particularly when there is structural dip. A pronounced directional permeability trend can control the arrangement of flooding patterns. The original waterflood pattern for the North Burbankfield was a five-spot with lines of injectors and producers oriented along the southwest/northeast direction.

It has east-west jointing or fracturing that leads to an effective permeability in the east-west direction that is five times the effective permeability in the north-south direction. When this geological feature was recognized, the waterflood was developed as a line drive by drilling in fill wells for injection, injecting water in east-west rows of wells, and producing alternate rows of production wells as shown in Fig. Although the surface arrangement of wells is a five-spot, the subsurface fluid movement approximates a line drive because of directional permeability. A sound program that couples geological Evaluation with engineering analysis often leads to better selection of flooding patterns and improved waterflooding performance.

5.1 Injection Rates

Injection rate is a key economic variable in the evaluation of a waterflood. When a waterflood is conducted in an established area, there may be data or correlations based on Operating experience. Typically, injection rates are correlated in terms of injectivities as barrels per day per acre foot, barrels per day per net foot of sand, or barrels per day per net foot per pounds per square inch. Specific values are dependent on reservoir rock properties, fluid/rock interactions, spacing, and available pressure drop. Comparable values would be expected under similar reservoir and operating conditions.

It is often possible to estimate injection rates from relatively simple equations when rates are not known. Two situations are of interest in waterflooding operations. If water injection is initiated before a mobile-gas saturation develops, the system may be treated as if it were liquid filled.

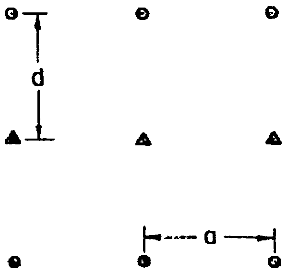
Another case is the depleted reservoir where a mobile-gas saturation develops during primary production by solution-gas drive. In these reservoirs, initial injection rates decline rapidly as the mobile gas is displaced. Because controlling rates are those for liquid-filled systems, we begin our discussion with these systems.

5.1.1 Exact expressions for injection rates in fully developed patterns at unit mobility ratio

Direct Line Drive

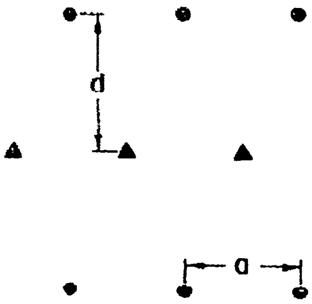
$$i = \frac{3.541\, kh(\Delta p)}{\mu\left(\ln \frac{a}{r_w} + 1.571\frac{d}{a} - 1.838\right)}$$

$$\frac{d}{a} \geq 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$



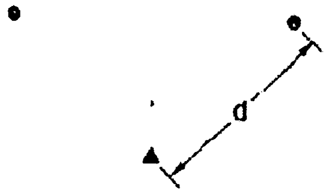
Staggered Line Drive

$$i = \frac{3.541\, kh(\Delta p)}{\mu\left(\ln \frac{a}{r_w} + 1.571\frac{d}{a} - 1.838\right)} \quad \ldots\ldots\ldots\ldots \quad (2)$$



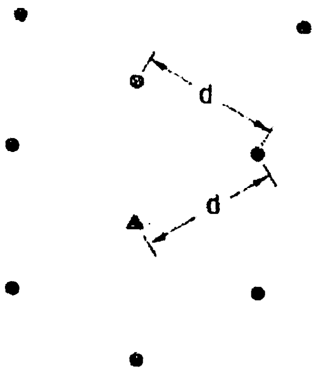
Five-Spot

$$i = \frac{3.541\, kh(\Delta p)}{\mu\left(\ln \frac{d}{r_w} - 0.619\right)} \quad \ldots\ldots\ldots\ldots \quad (3)$$



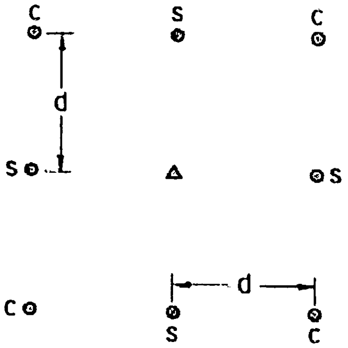
Seven-Spot

$$i = \frac{4.72\, kh(\Delta p)}{\mu\left(\ln \frac{d}{r_w} - 0.569\right)} \quad \ldots\ldots\ldots\ldots \quad (4)$$



Nine-Spot

$$i = \frac{3.541\ kh(\Delta p)_{i,c}}{\frac{1+R}{2+R}\left(\ln \frac{d}{r_w} - 0.272\right)\mu} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$



$$i = \frac{7.082\ kh(\Delta p)_{i,s}}{\left[\frac{3+R}{2+R}\left(\ln \frac{d}{r_w} - 0.272\right) - \frac{0.693}{2+R}\right]\mu} \quad \ldots\ldots\ldots \quad (6)$$

R = ratio of producing rate of corner well to side well,

$(\Delta p)_{i,c}$ = pressure difference between injection well and corner well, and

$(\Delta p)_{i,s}$ = pressure difference between injection well and side well.

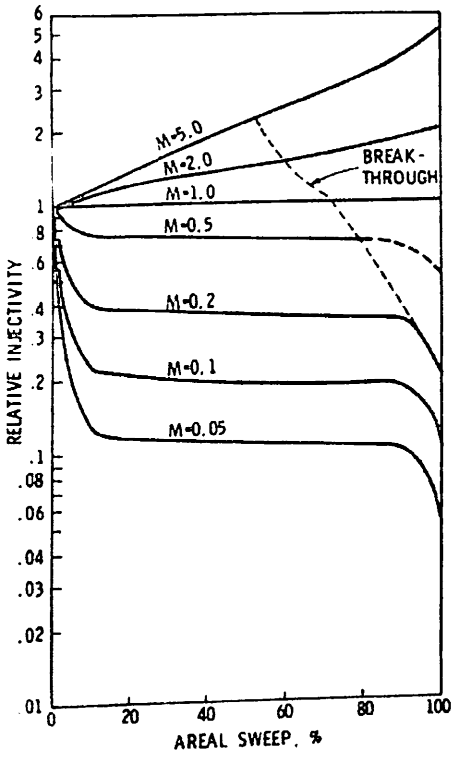
5.1.2 Injection Rates for Pattern Floods in Liquid-Filled Systems-M ≠1

Most floods do not have unit mobility ratios, and thus injection rates change with the area of the pattern that is swept. Fig shows a correlation equation 7 for the change in the conductance with area swept *(EA)* and mobility ratio, *M,* for miscible displacement in a five-spot pattern from the data of Caudle and Witte

The conductance or relative injectivity defined by Equation 7

$$\gamma = \frac{i}{i_b}. \quad \ldots\ldots(7)$$

γ is the ratio of the injection rate during miscible displacement to the injection rate computed from Equation. 3 assuming the displaced phase is flowing in the pattern as a single phase under the same pressure drop.



Correlation of conductance ratio (relative injectivity) with areal sweep efficiencies at selected mobility ratios for miscible displacement in five spot pattern

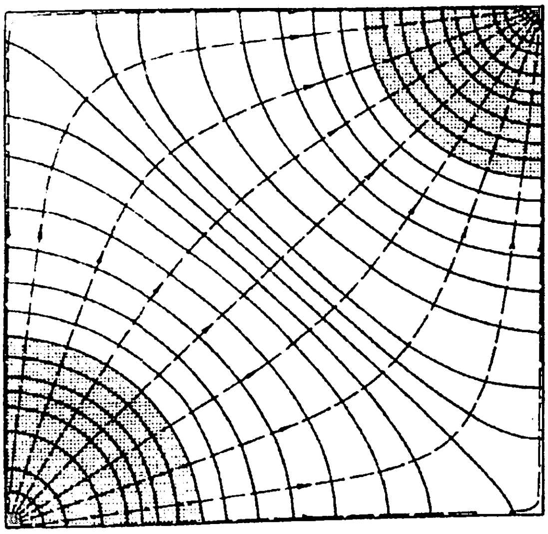
mobility ratio for a miscible displacement, *M*, process is

$$M = \frac{\left(\frac{k}{\mu}\right)_d}{\left(\frac{k}{\mu}\right)_D}$$

$$M = \frac{\mu_d}{\mu_D}.$$

Injection rates in a five-spot pattern are fairly constant after the displacing front moves away from the injection wellbore. The large change in conductance at small values of *EA* is because much of the flow resistance is found in the region immediately around the injection well. A similar rapid change in injection rate is observed as *EA* approaches 100% as a result of changes in the flow resistance at the producing well. Immiscible displacement processes involve the flow of two phases as well as the presence of residual saturations of one or more phases in swept and un swept portions of the reservoir. In the next section, a general approach to estimate injection rates in an immiscible displacement process for uniform and non uniform patterns is developed.

Injection rates can be estimated from models that approximate fluid flow in pattern floods. These models rely on characteristics of fluid movement near injection and production wells for homogeneous systems that can be seen in Fig.



Isopotentials and streamlines in a quadrant of homogeneous five-spot pattern.

Fig. shows isopotentials for a quadrant of a five-spot when the mobility ratio is I.O- i.e., single phase flow. Flow is radial in about 23% of the pattern area around the production and injection wells. About 90% of the potential drop (or pressure drop) occurs in this region. A similar situation exists for other flooding patterns-such as the direct line drive and nine-spot.

In a waterflood, a radial flood front forms when water is injected into a well, even when the mobility ratio is not unity. Injection rates change because the flow resistance behind the flood front either increases or decreases depending on the mobility of the fluids in the displaced region. Although the fraction of the pattern area where the fluid flow is radial varies with mobility ratio, the flow resistance for a significant portion of the flood occurs in the radial flow region

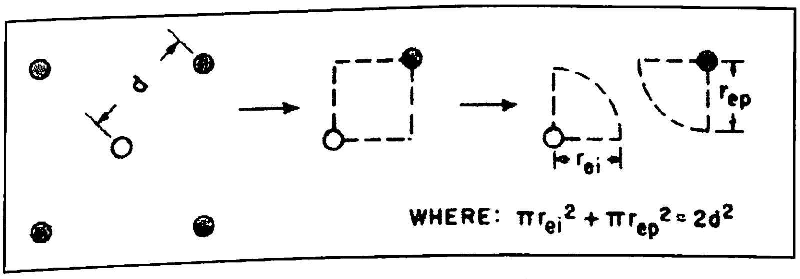
When relative permeability curves and fluid properties fall within certain ranges, the displacement process is piston-like-that is, the breakthrough or flood-front saturation is essentially 1-*Sor* and there is little oil displacement in the swept area after the flood front arrives. Piston-like displacement occurs whenever a straight line can be drawn on the fractional flow curve from the point representing initial water saturation to 1- S*or* without intersecting the curve. In piston displacement, the mobility ratio is given by Equation.

$$M=\frac{\left(\frac{k_{rw}}{\mu_w}\right)_{S_{or}}}{\left(\frac{k_{ro}}{\mu_o}\right)_{S_{iw}}}$$

Data to compute this mobility ratio are usually obtained from flood pot tests because *(krw) Sor* and *(kro) S*iw are the endpoints of the relative permeability curves.

The method for calculating injection rates approximates the pattern area with radial sections or a combination of radial and linear sections whose surface area is equal to the pattern area.4 Injection rates in the pattern are estimated by combining radial and linear flow equations. This method is illustrated with the development of an approximate equation for the injection rate in a five-spot pattern For *M*≠ 1 Then; the method will be extended for use with other patterns-such as a line drive.

5.2 Approximate Model for Five-Spot.



Approximate model for fluid flow in a five-spot pattern.

Fig.5.2a shows one-fourth of a five-spot approximated with two segments of a circle that have radii *rep* and *rei,* respectively. According to our model assumptions, the pattern area is equal to the area of the approximating flow segments. Thus, when *rei=rep=re,* then *re=d√2.* The fluid-flow model assumes that flow is steady, incompressible, and radial from the injection well to *rei,* the outer radius of the injection segment; then fluid flows radially from the outer radius of the production segment, *rep'* to the production well. The equivalent flow system is depicted in Fig. 5.2b

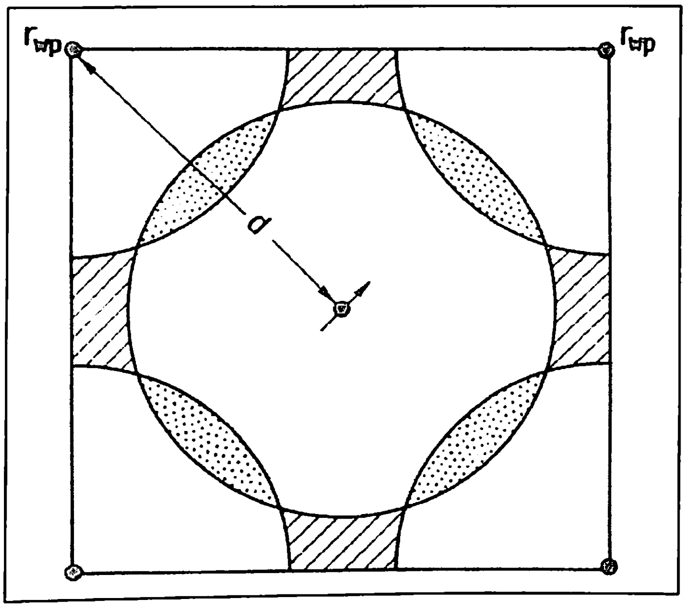


Fig 5.2 b

The location of the flood-front saturation, *rf,* can be computed with material balance. Neglecting the radius of the wellbore,

$$W_i=(\pi r_f^2\phi h)(\bar{S}_{wf}-S_{iw}). \quad \ldots\ldots \qquad (8)$$

Thus

$$r_f=\sqrt{\frac{W_i}{\pi\phi h(\overline{S_{wf}}-S_{iw})}}, \quad \ldots\ldots\ldots \qquad (9)$$

for $rf < re$, when the displacement is piston-like, *Swf= l-Sor'* Then,

$$r_f=\sqrt{\frac{W_i}{\pi\phi h(S_{oi}-S_{or})}}, \quad \ldots\ldots\ldots \qquad (10)$$

when Siw = 1 - S*oi.*

The expression for the flow rate is obtained by considering the system as resistances in series. For steady state radial flow in a porous rock with radii *rf* and *rz,*

$$p_1 - p_2 = \frac{i \ln \frac{r_2}{r_1}}{\left(\frac{k}{\mu}\right) 2\pi h} \quad \ldots\ldots\ldots \quad (11)$$

$$p_w - p_p = (p_w - p_f) + (p_f - p_e) + (p_e - p_p) \quad \ldots \quad (12)$$

Or

$$p_w - p_p = \frac{i \ln \frac{r_f}{r_w}}{\left(\frac{k_w}{\mu_w}\right)_{S_{or}} 2\pi h} + \frac{i \ln \frac{r_e}{r_f}}{\left(\frac{k_o}{\mu_o}\right)_{S_{iw}} 2\pi h} + \frac{i \ln \frac{r_e}{r_{wp}}}{\left(\frac{k_o}{\mu_o}\right)_{S_{iw}} 2\pi h} . \quad \ldots\ldots(13)$$

Solving for the injection rate,

$$i = \frac{2\pi h (p_w - p_p)}{\frac{\ln \frac{r_f}{r_w}}{\lambda_w} + \frac{\ln \frac{r_e}{r_f}}{\lambda_o} + \frac{\ln \frac{r_e}{r_{wp}}}{\lambda_o}} , \quad \ldots \quad \ldots\ldots.(14)$$

And

$$i = \frac{2\pi \lambda_o h (p_w - p_p)}{\frac{1}{M} \ln \frac{r_f}{r_w} + \ln \frac{r_e}{r_f} + \ln \frac{r_e}{r_{wp}}} . \quad \ldots\ldots\ldots (15)$$

Equation 15 may be compared with the exact solution (Equation 3) at M= I to check the effects of the assumptions used in developing the model. When M= 1 and *rw* = *rp,* Equation 16 becomes

$$i = \frac{2\pi \lambda_o (p_w - p_p)}{\ln \left(\frac{r_e^2}{r_w^2}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Because

$$r_e = \frac{d}{\sqrt{\pi}} ,$$

Equation. 16 may be written in terms of *d* as in Eq.17:

$$i=\frac{\pi\lambda_o h(p_w-p_p)}{\ln\left(\frac{d}{r_w}\right)-0.572}. \quad \text{.......................} (17)$$

If variables are expressed in oilfield units (pounds per square inch, centipoise, days, feet, barrels per day, darcies), the corresponding relationship is Equation 18.

$$i=\frac{3.541\lambda_o h\Delta p}{\ln\left(\frac{d}{r_w}\right)-0.572}. \quad \text{..................} (18)$$

It is useful to express the injection rate given by Eq.18 in terms of a conductance to compare the effects of the mobility ratio on injection rates. 8 The conductance, 'γ' defined by Equation. 19 is the ratio of the injection rate during the flood to the injection rate if oil were injected into the formation at the same pressure drop.

$$\gamma=\frac{i}{i_b}=\frac{2\left[\ln\left(\frac{d}{r_w}\right)-0.572\right]}{\frac{1}{M}\ln\frac{r_f}{r_w}+\ln\frac{r_e}{r_f}+\ln\frac{r_e}{r_w}}. \quad \text{............} (19)$$

Because

$$E_A=\frac{\pi(r_f^2-r_w^2)}{2d^2} \quad \text{.............................} (20)$$

and

$$r_f=d\sqrt{\frac{2E_A}{\pi}} \quad \text{..................................} (21)$$

when *rf>>rw,* and after rearranging Equation. 19

$$\gamma=\frac{2\left[\ln\left(\frac{d}{r_w}\right)-0.572\right]}{\frac{1}{M}\ln\left(\frac{d\sqrt{2E_A/\pi}}{r_w}\right)+\ln\left(\frac{d}{r_w\sqrt{2E_A/\pi}}\right)} \quad \ldots(22)$$

for $E_A \leq 0.5$. The conductance can be expressed also in terms of *Wi* with Eq.8.

Fig illustrates the change in conductance with mobility ratio *(M)* and *E A* for a waterflood in a 10-acre [40 469-m2] five-spot, where *d=467* ft [142 m] and *r w*=0.5 ft [0.15 m]. The computations show that most of the rate changes occur when the areal sweep is less than 0.1. Thereafter, the conductances remain relatively constant to *EA* =0.5-the limit of the model. Effects of mobility ratio on injection rates can be observed early in the waterflood. Thus injection rates based on short pilot tests or early waterflood performance can be misleading when projected to the entire flood.

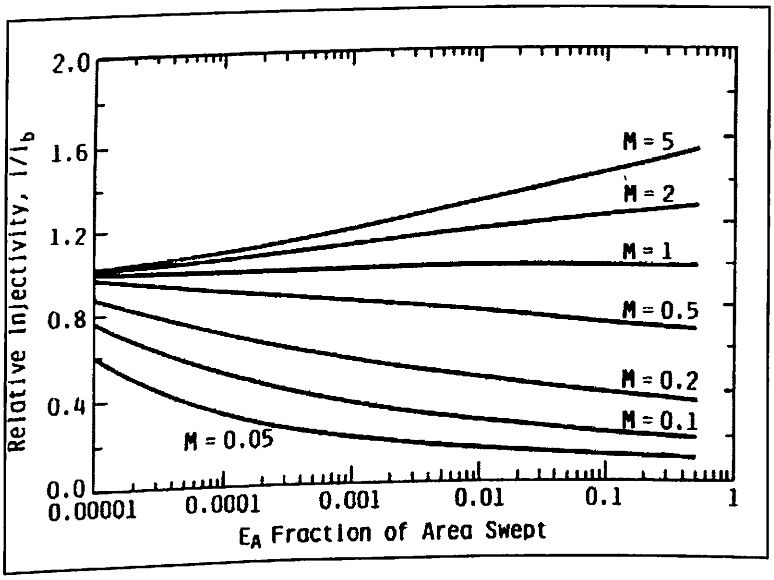


Fig 5.2c

Variation of conductance (i/ib) with mobility ratio.

5.3 Depleted Reservoirs

Waterfloods are frequently initiated after some of the oil has been produced by solution-gas drive. Mobile-gas saturation is present when water injection begins. When there is an initial gas saturation and sufficient oil saturation for an oil bank to form, the displacement process is represented by Fig.

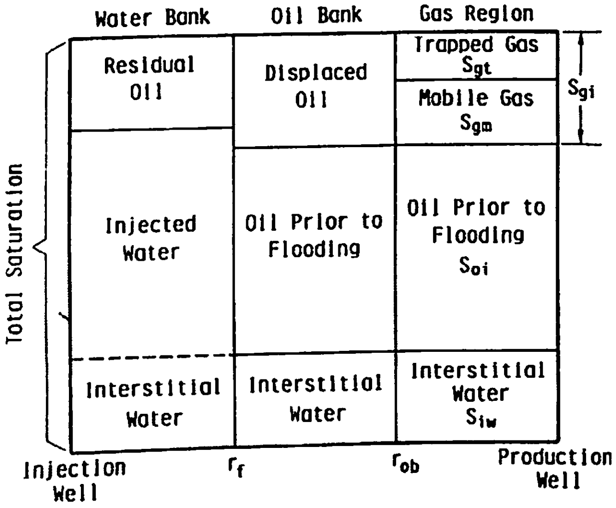


Fig 5.4a

Saturation distribution during the waterflood of a depleted reservoir when trapped gas redissolves

The oil bank displaces the mobile gas, leaving a trapped-gas saturation, S*gt*. At usual waterflood pressures, the trapped-gas saturation redissolves in the oil. During the fill-up period, mobile gas is displaced by the oil bank. Gas mobility is large in the region ahead of the oil bank. There is little pressure drop between the oil bank and the producing well; consequently, oil production is negligible.

Fluid movement in the oil and water banks during the fill-up period is radial until the oil banks from adjacent patterns meet or "interfere" with each other. Fig. illustrates the position of the oil bank in a five-spot pattern at interference

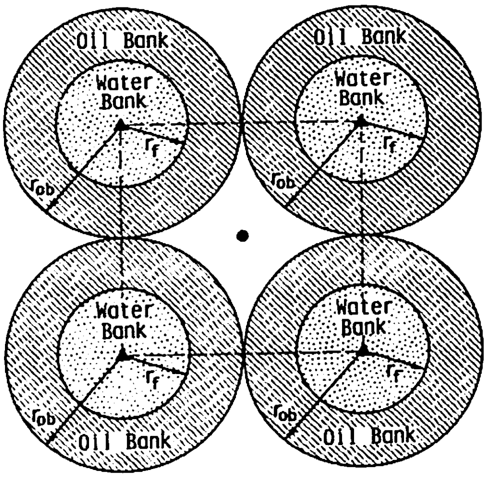


Fig5.3 (b)
interference of oil banks in the waterflood of a five-spot pattern that has a uniform initial gas saturation

Eq.23 describes the injection rate during fill up to the interference point for a five-spot when the flow resistance is neglected ahead of the oil bank.

$$i=\frac{2\pi\lambda_o(p_{wi}-p_{wp})}{\frac{1}{M}\ln\left(\frac{r_f}{r_w}\right)+\ln\left(\frac{r_{ob}}{r_f}\right)},$$
...............(23)

where

rob = radius of the oil bank, *r*w ≤*rob* ≤*d*√2, and

rf = radius of the flood-front saturation.

Both *rob* and *rf* may be defined by a material balance on the injected water

$$W_i=\pi(r_f^2-r_w^2)(\bar{S}_w-S_{iw})h\phi.$$
.........(24)

Solving for *rf* gives

$$r_f=\sqrt{\frac{W_i}{\pi\phi h(\bar{S}_w-S_{iw})}+r_w^2}$$
............(25)

The volume of water injected to fill-up is equal to the volume of gas displaced by the oil bank as the initial gas saturation, S*gi*, is reduced to the trapped-gas saturation, n *Sgt*. A material balance yields Eq.26 for *rob:*

$$r_{ob}=\sqrt{\frac{W_i}{\pi\phi h(S_{gi}-S_{gt})}+r_w^2}\ .$$
............(26)

At interference, $r_{ob}=d\sqrt{2}$. Equation 26 can be used to compute the volume of water required to reach interference. If the injection rate is constant, the interference time can be computed from W_i' Neglecting the wellbore radius

$$W_{ii}=\frac{\pi d^2}{2}\phi h(S_{gi}-S_{gt}). \qquad \text{...... (27)}$$

The volume of water injected at fill-up is given by Equation 28.

$$W_{if}=2d^2\phi h S_{gi}, \qquad \text{...............(28)}$$

when the trapped gas is redisolved in the oil with negligible change in volume. At fill-up, r_f is obtained by substituting Equation 28 into Equation 25 to obtain

$$r_f=\sqrt{\frac{2d^2 S_{gi}}{\pi(\bar{S}_{w'}-S_{iw'})}+r_w^2}\ . \qquad \text{......(29)}$$

If $r_f<d$ for a five-spot, the injection rate after fill-up is given by Eqs. 17 and 18.

6.0 Example for cumulative water injection calculation with 5 spot pattern and depletion drive

An estimate of the injection rate is needed for a waterflood in a 5-acre [20 235-m^2] five-spot pattern. The reservoir has been depleted by solution-gas drive and has an initial gas saturation of 0.10. From laboratory correlations, the gas saturation trapped by the oil bank is estimated to be 0.04. Flood pressure is expected to cause the trapped gas to redissolve in the oil, leaving the residual gas saturation equal to zero. Reservoir and fluid/rock properties are given in Table. A pressure difference of 330 psi [2.3 MPa] is maintained between the injection well and the production well.

RESERVOIR AND FLUID/ROCK PROPERTIES	
Pattern area, acres	5
Thickness, ft	15
Radius of injection and production wells, ft	0.5
Siw	0.27
So,	0.30
Porosity	0.15
Permeability to liquid (base permeability), darcies	0.203
kro at *Siw*	0.7
krw (at *So,)*	0.15
Viscosity of oil, cp	2.0
Viscosity of water, cp	1.0

Solution. Injection rates can be estimated with the approximate model until interference, $(r_{ob} \leq d\sqrt{2})$, where d is the distance between the injection well and the production well. When $r_{ob} \geq d\sqrt{2}$, flow ceases to be radial and the oil-bank flood front moves at different velocities toward the production well. Because the point of interference determines the last time when the approximate model is valid, computations begin there. The distance between the injection well and the production well is computed as follows.

$$d = \sqrt{\{(5 \text{ acres} * 43560 \text{ sq ft/acres})/2\}}$$

$$= 330 \text{ ft}$$

The interference occurs when $rob = d\sqrt{2}$ or 233.33 ft [71.12 m]. From Eq.37, the volume of water injected at interference is

$$W_{ii} = \frac{\pi}{2}(330 \text{ ft})^2(0.15)(15 \text{ ft})(0.10)$$

$$= 38{,}488 \text{ cu ft}$$

$$= 6{,}855 \text{ bbl.}$$

Equation 23 is used to compute the injection rate for O≤ *Wi* ≤ 6,855 bbl [1090 m3]. We assume *Sw=1.0 - Sor=0.7.* From Equation 35,

$$r_f=\sqrt{\frac{(6{,}855)(5.615)}{\pi(0.15)(15.0)(0.70-0.27)}+(0.5)^2}$$

$$=112.5 \text{ ft.}$$

In oilfield units (darcies, centipoise, feet, pounds per square inch, and barrels per day), Equation 23 is

$$i=\frac{7.082k_b\left(\frac{k_{ro}}{\mu_o}\right)h(p_{wi}-p_{wp})}{\frac{1}{M}\ln\frac{r_f}{r_w}+\ln\frac{r_{ob}}{r_f}}.$$

The mobility ratio, *M,* is computed from the endpoints of the relative permeability curves.

$$M=\frac{\left(\frac{k_{rw}}{\mu_w}\right)_{S_{or}}}{\left(\frac{k_{ro}}{\mu_o}\right)_{S_{iw}}}=\frac{\left(\frac{0.15}{1.0}\right)}{\left(\frac{0.7}{2.0}\right)}=0.43.$$

Substituting into Equation 40 gives *i* at interference

$$i=\frac{(7.082)(0.203)\left(\frac{0.7}{2.0}\right)(15)(330)}{\frac{1}{0.43}\ln\frac{112.5}{0.5}+\ln\frac{233.3}{112.5}}=\frac{2{,}491}{13.32}$$

$$=186.9 \text{ B/D}$$

Injection rates from the beginning of injection to interference can be computed easily. Table contains injection rates for 10 equal increments of *Wi,* 0≤*Wi*≤*Wii* The initial injection rate is quite high (2,300 BID [366 m3/dD but declines rapidly with *Wi* as the space occupied by gas is filled with water and oil banks. High initial injection rates that decline rapidly (or lower rates at small pressure drops) should be expected in a waterflood of a depleted reservoir.

INJECTION RATES TO INTERFERENCE				
Wi (bbl)	*r*, (It)	*Rob* (ft)	*i* *(B/D)*	Time (days)
0.1	0.66	1.02	2,298	0.00
686.5	36.59	73.79	233.2	0.54
1,370.9	50.33	104.36	216.8	3.59
2,056.4	61.64	127.81	208.2	5.81
2,741.8	71.17	147.58	202.5	10.15
3,427.3	79.57	166.00	198.3	13.57
4,112.7	87.17	180.75	196.0	17.06
4,798.2	94.15	196.23	192.3	20.60
5,483.7	100.65	208.71	190.0	24.18
6,169.1	106.76	221.37	188.1	27.81
6,854.6	112.53	233.35	186.3	31.47

Continued injection of water leads to fill-up of the gas space. The volume of water injected to fill-up is obtained from Equation 28.

$$W_{if}=\frac{(2)(330)^2(0.15)(15)(0.10)}{5.615}$$

$$=8{,}728 \text{ bbl.}$$

At fill-up, the radius of the water bank is given by Equation 25.

$$r_f=\sqrt{\frac{(8{,}728)(5.615)}{\pi(0.15)(15)(0.70-0.27)}+(0.5)^2}$$

$$= 127 \text{ ft.}$$

The entire pattern is 100% liquid-saturated so that Equation 7 now describes the injection rate from fill-up to the radius where the assumptions made in developing Eq.15 are not valid-that is, $rf=d\sqrt{2}$ is the maximum value of *rf i.e. re* where Equation 15 is valid. Thus

$$\mathbf{re = 330/\sqrt{\pi}}$$

$$= 186.2 \text{ ft.}$$

Solving for *Wi* with Equation 24 gives

$$W_i = \frac{\pi(186.2)^2(0.70-0.27)(15)(0.15)}{5.615}$$

=18,768 bbl.

Therefore, the injection rate for 8,728≤*Wi*≤18,768 may be estimated from Eq.15.

Applying Equation 15 with *re=186.2* ft [56.8 m] and *rf*= 127.0 ft [39 m] gives the injection rate at the instant fill-up occurs. Thus

$$i_f = \frac{(7.082)(0.203)(0.7/2.0)(15)(330)}{\frac{1}{0.43}\ln\left(\frac{127.0}{0.5}\right)+\ln\left(\frac{186.2}{127.0}\right)+\ln\left(\frac{186.2}{0.5}\right)} = \frac{2{,}491}{19.18}$$

$$=129.9 \text{ B/D.}$$

Between interference and fill-up, the injection rate decreased from 186.9 to 129.9 BID [29.7 to 20.6 m3/d] as *Wi* changed from 6,855 to 8,728 bbl [1090 to 1388 m3]. The time between interference and fill-up corresponds to the injection of 1,873 bbl [298 m3] of water at a declining rate.

As noted previously, the approximate model for the liquid-filled system applies for $Rf \leq d\sqrt{\pi}$. At $rf=d\sqrt{2}$,

$$i = \frac{2{,}491}{\frac{1}{0.43}\ln\frac{186.2}{0.5}+\ln\frac{186.2}{0.5}}$$

=126.5 B/D.

7.0 Evaluating the Assumption of Piston-Like Displacement.

In this section an investigation is made of when the total mobility ratio, M_t, should be used to estimate injection rates. Relative permeability data are required for the computation of M_t. For the purposes of this discussion, assume that below equations describe the relative permeability relationship in terms of the dimensionless water saturation, S_{wD}

$$k_{ro} = \alpha_1 (1 - S_{wD})^m$$

and

$$k_{rw} = \alpha_2 S_{wD}{}^n .$$

Where

$$S_{wD} = \frac{S_w - S_{iw}}{1 - S_{or} - S_{iw}} .$$

The mobility ratio based on the endpoints of the relative permeability curves is given by Equation [redacted]:

$$M = \left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{\mu_o}{\mu_w}\right) . \qquad \text{........(30)}$$

8.0 Computer Models for Design of Waterfloods

A computer model is necessary for waterflood design when the assumptions required to use approximate models or correlations based on scaled-model results cause large uncertainties in the estimated flood performance. The model helps the process of flood pattern design more accurate and efficient.

The level of uncertainty that is acceptable is a matter of engineering and management judgment and will not be addressed here.

This section introduces the capabilities of computer models to assist in the design of a water flood. Model is described in terms of the types of problems for which they are best suited and generally persist in the industry. Our emphasis is on identifying key factors of the flooding design and gives the values of crucial parameters.

Computer models are separated into two categories in this text. The stream tube model presented is a computer model in the sense that access to a computer is necessary to use the model.

All other computer models that are available in the industry are referred to as numerical simulators because they are based on the numerical solution of the partial differential equations describing two- and three-phase flow in a porous rock in one, two, or three spatial coordinates. In this section, some of the capabilities of computer models to assist in the design of a waterflood are presented.

9.0 Conclusion

1. Project basically deals with calculation of percentage recovery, and cumulative water injected within the reservoir which is on water flooding. by use of Buckley-Leverett one dimensional displacement and fractional flow equation

2. For this calculations the formulations are being converted into a computer model and by use of basic input data like initial reservoir water saturation (swi) oil viscosity (μo) water viscosity (μw), avg. thickness of reservoir (h, ft) porosity (φ) we are able to generate values Fractional Flow (fw) of % recovery (RF) and total volume of water injected (wi) within the reservoir at various water saturation values.

3. The data of Sw vs. Fw generated from the computer model can also be used directly to develop and fractional flow curve from where percent recovery can be calculated graphically also. Percent recovery varies according to the injection pattern being used in the reservoir our calculations on examples are based on five spot injection pattern having recovery efficiency of 65%,although program can be altered for use of different injection patterns efficiencies

4. Total Volume of water injected (Wi) within the reservoir is been calculated at different percentage of Sw and at last the final volume is when value of water saturation equals

5. The reservoir is considered to be having same permeability i.e. homogenous in nature and we are considering flow to be one dimensional in nature also the injection rate (i) is considered to be constant

6. The data generated from the computer model are compared and matched to check the validity of the model with basic graphical calculations done normally for calculations of Fw, % recovery and Water injection.

10.0 Project Limitations

1. Computer model generated can only work considering the reservoir to be homogeneous in nature i.e. having same relative and absolute permeability throughout the reservoir area.

2. The water injection rate(i) is considered to be constant for a single reservoir from the time of injection to water breakthrough and production, although in advance modeling the injection rate varies as water front advances till it reaches the oil bank

3. The flow is considered to be one dimensional and thus basic concept of Buckley-Leverett one dimensional displacement is considered in the computer model. Although modern numerical simulators can generate data for 3-dimensional flow and reservoir.

4. Due to basic restriction of high end programming know about, the model only deals with reservoir with water flooding having no gas influx or gas drive also the mobility ratio calculations are not included in the model. Although an example is produced in the text which deals with water flooding in a depletion drive reservoir under a five spot pattern of water flooding.

5. The advance modeling that can be done for more acute results of water flooding of reservoir are discussed as theoretical plan in the text above.

6. The computer software that has been developed is not much user friendly.

7. The data generated by the software is imported to Microsoft Excel for interpretation. This process is quite time consuming.

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Books

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4. SPE monograph volume 3 Craig Jr. Forest F. "*The Reservoir Engineering aspects of Water flooding*".
5. SPE Monograph Series "*Water Flooding*"
6. Lafore Robert "*Object Oriented Programming in C++*"
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Appendix A-1

Program to calculate

- ✓ Fractional Flow,
- ✓ Percentage recovery
- ✓ Water break through point

With help of basic input data of initial water saturation (S_{wi}), Viscosity of Oil(μ_o),and viscosity Of water(μ_w) considering the water injection pattern to be 5 spot

There are two cases in the program, considering the reservoir to be

- ✓ unconsolidated sand well sorted
- ✓ cemented sand stone oolitic limestone,vuglar rocks

for case 1: Reservoir to be unconsolidated sand well sorted

Soultion(data sheet as pr program)

```
Menu enter your choice accordingly:
        Enter 1. For unconsolidated sand well sorted
         Enter 2. For Cemented Sand Stone Oolitic Limestone,Vuglar Rocks


Enter Choice : 1
 Enter Value Of Initial Water Saturation : 0.20
 Enter Oil Viscosity: 5 cp
 Enter Water Viscosity: 0.5 cp
```

Sw	So	S*	Krw	Kro	Fw
0.2	0.8	1	0	1	0
0.25	0.75	0.9375	0.000244	0.823975	0.002954
0.3	0.7	0.875	0.001953	0.669922	0.028329
0.35	0.65	0.8125	0.006592	0.536377	0.109445
0.4	0.6	0.75	0.015625	0.421875	0.27027
0.45	0.55	0.6875	0.030518	0.324951	0.484309
0.5	0.5	0.625	0.052734	0.244141	0.683544
0.55	0.45	0.5625	0.08374	0.177978	0.824718
0.6	0.4	0.5	0.125	0.125	0.909091
0.65	0.35	0.4375	0.177979	0.08374	0.955064
0.7	0.3	0.375	0.244141	0.052734	0.978857
0.75	0.25	0.3125	0.324951	0.030518	0.990696
0.8	0.2	0.25	0.421875	0.015625	0.99631
0.85	0.15	0.1875	0.536377	0.006592	0.998773

0.9	0.1	0.125	0.669922	0.001953	0.999708
0.95	0.05	0.0625	0.823975	0.000244	0.99997
1	0	0	1	0	1

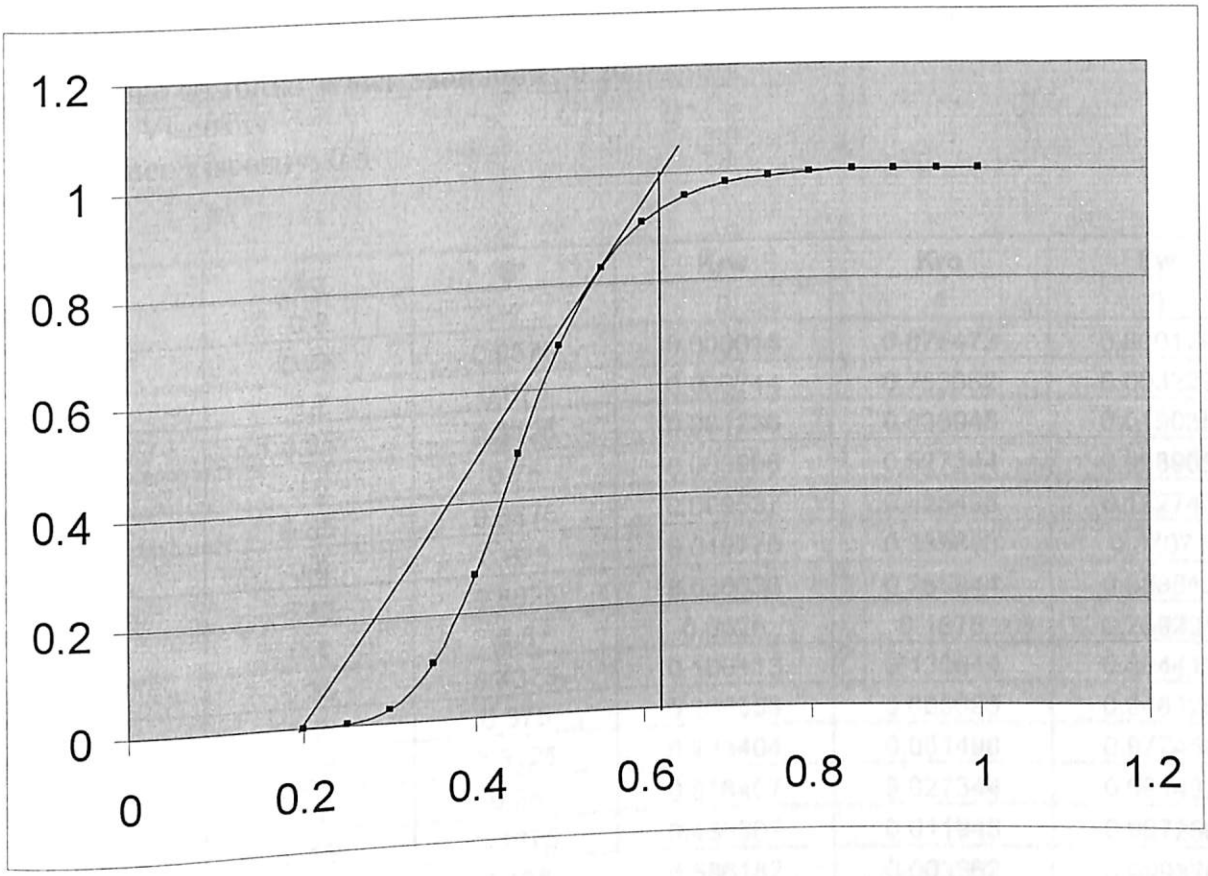
Water Break through S_{wbt}= **0.63096**

Sw	dsw/dfw
0.2	0
0.25	0
0.3	0.825226
0.35	2.68439
0.4	4.42879
0.45	4.56534
0.5	3.39146
0.55	2.07147
0.6	1.14125
0.65	0.5954
0.7	0.299038
0.75	0.143793
0.8	0.06452
0.85	0.025459
0.9	0.007558
0.95	0.000594

% W C	Water sat.	Sw-Swi	(Swi-Swi)/Swi	R.F
0	0.63096	0.43096	0.5387	0.350155
10	0.667864	0.467864	0.58483	0.380139
20	0.704768	0.504768	0.63096	0.410124
30	0.741672	0.541672	0.67709	0.440108
40	0.778576	0.578576	0.72322	0.470093
50	0.81548	0.61548	0.76935	0.500077
60	0.852384	0.652384	0.81548	0.530062
70	0.889288	0.689288	0.86161	0.560046
80	0.926192	0.726192	0.90774	0.590031
90	0.963096	0.763096	0.95387	0.620016
100	1	0.8	1	0.65

To check the validity we generated the Flow curve separately and checked the Breakthrough saturation

Generating Fractional Flow Curve Sw (x Axis) vs Fw (y Axis)



Both Break through Saturations consides with little Deviations for case 1

Appendix A-2

For 2nd case: Reservoir is cemented sand stone oolitic limestone,vuglar rocks

Solution

```
Menu enter your choice accordingly:
Enter 1. For Unconsolidated Sand Well Sorted
Enter 2. For Cemented Sand Stone Oolitic Limestone,Vuglar Rocks
Enter Choice : 2

Enter Value Of Initial Water Saturation : 0.20
Enter Oil Viscosity: 5
Enter Water Viscosity: 0.5
```

Sw	So	S*	Krw	Kro	Fw
0.2	0.8	1	0	1	0
0.25	0.75	0.9375	0.000015	0.875473	0.000174
0.3	0.7	0.875	0.000244	0.753662	0.003229
0.35	0.65	0.8125	0.001236	0.636948	0.019035
0.4	0.6	0.75	0.003906	0.527344	0.068966
0.45	0.55	0.6875	0.009537	0.426498	0.182743
0.5	0.5	0.625	0.019775	0.335693	0.37071
0.55	0.45	0.5625	0.036636	0.255844	0.588813
0.6	0.4	0.5	0.0625	0.1875	0.769231
0.65	0.35	0.4375	0.100113	0.130844	0.884411
0.7	0.3	0.375	0.152588	0.085693	0.946826
0.75	0.25	0.3125	0.223404	0.051498	0.977468
0.8	0.2	0.25	0.316407	0.027344	0.991432
0.85	0.15	0.1875	0.435807	0.011948	0.997266
0.9	0.1	0.125	0.586182	0.003662	0.999376
0.95	0.05	0.0625	0.772477	0.000473	0.999939
1	0	0	1	0	1

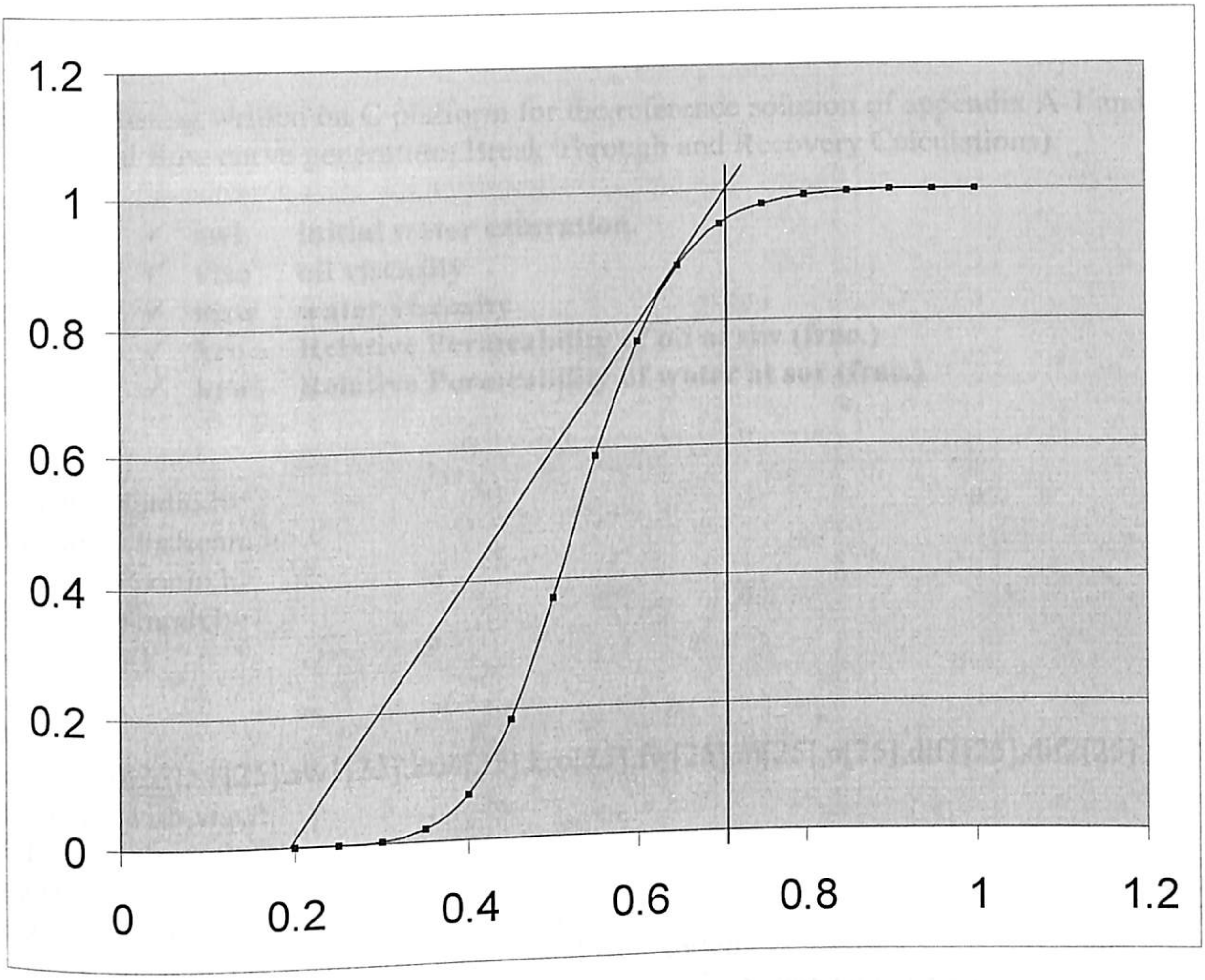
Water Break through S_{wbt}= **0.714017**

Sw	dsw/dfw
0.2	0
0.25	0
0.3	0
0.35	0.360136
0.4	1.53367
0.45	3.45796
0.5	4.7389
0.55	4.26075
0.6	2.83124
0.65	1.56605
0.7	0.779598
0.75	0.360589
0.8	0.153924
0.85	0.057657
0.9	0.01628
0.95	0.001225

%wc	Water Sat	Sw-Swi	(Swi-Swi)/Swi	R.F
	0	0.714017	0.514017	0.642521
10	0.742615	0.542615	0.678269	0.440875
20	0.771214	0.571214	0.714017	0.464111
30	0.799812	0.599812	0.749765	0.487347
40	0.82841	0.62841	0.785513	0.510583
50	0.857009	0.657009	0.821261	0.53382
60	0.885607	0.685607	0.857009	0.557056
70	0.914205	0.714205	0.892757	0.580292
80	0.942804	0.742803	0.928504	0.603528
90	0.971402	0.771402	0.964252	0.626764
100	1	0.8	1	0.65

To check the validity we generated the Flow curve separately and checked the Breakthrough saturation

Generating Fractional Flow Curve Sw (x Axis) vs Fw (y Axis)



Both Break through Saturations consides with little Deviations for case 2 also

Appendix A-3

Program listing written on C platform for the reference solution of appendix A-1 and A-2 (Fractional flow curve generation, Break Through and Recovery Calculations).

✓	swi	**initial water saturation.**
✓	viso	**oil viscosity**
✓	visw	**water viscosity**
✓	kro	**Relative Permeability of oil at siw (frac.)**
✓	krw	**Relative Permeability of water at sor (frac.)**

```
# include<stdio.h>
# include<iostream.h>
# include<conio.h>
# include<math.h>
void main()
{
float
sw[25],so[25],s1[25],sw1[25],krw[25],kro[25],fw[25],m[25],n[25],dif1[25],dif2[25];
float swi, viso,visw;
// for two phase flow...
int ch,a,b;
cout<<" \n Menu enter your choice accordingly: ";
cout<<" \n enter 1. for unconsolidated sand well sorted ";
cout<<" \n enter 2. for cemented sand stone oolitic limestone,vuglar rocks ";
cout<<" \n enter choice : ";
cin>> ch;
if(ch==1)
{
 cout<<" \n enter value of initial water saturation :";
 cin>>swi;
 cout<<"\n enter oil viscosity: ";
 cin>>viso;
 cout<<"\n enter water viscosity: ";
 cin>>visw;
 for ( a=1;a<=20;a++)
 {  if(a==1)
     {sw[a]=swi;}
    if(a>1)
     {sw[a]=sw[a-1]+0.05;}
    so[a]=1-sw[a];
    s1[a]=so[a]/(1-swi);
    sw1[a]=(sw[a]-swi)/(1-swi);
    krw[a]=pow (sw1[a],3);
    kro[a]=pow((so[a]/(1-swi)),3);
```

```
   if (krw[a]==0)
       {fw[a]=0;}
   if (krw[a]>0.0)
       {
         m[a]=(visw*kro[a])/(krw[a]*viso); /* mobility ratio cal*/
        n[a]=m[a]+1;
         fw[a]=1/n[a];
       }
                                   }
cout<<"Sw\t"<<"So\t"<<"  S*\t"<<"  S*w\t"<<"    Krw\t"<<" Kro\t"<<"    Fw\t"<<"\n";
for( a=1;a<=10;a++)
{
  cout<<sw[a]<<"\t"<<so[a]<<"\t";
  printf("%f ",s1[a]);
  cout<<" ";
  printf("%f ",sw1);
  cout<<" ";
  printf("%f ",krw[a]);
  cout<<" ";
  printf("%f ",kro[a]);
  cout<<" ";
  printf("%f ",fw[a]);
  cout<<"\n";
}
}
if(ch==2)
{
cout<<" \n enter value of initial water saturation :";
cin>>swi;
cout<<"\n enter oil viscosity: ";
cin>>viso;
cout<<"\n enter water viscosity: ";
cin>>visw;
for ( a=1;a<=10;a++)
{if(a==1)
 {sw[a]=swi;}
 if(a>1)
 {sw[a]=.15*a;}
 so[a]=1-sw[a];
s1[a]=so[a]/(1-swi);
sw1[a]=(sw[a]-swi)/(1-swi);
krw[a]=pow (sw1[a],4);
float d= (pow( so[a],3))*(2*sw[a]+so[a]-2*swi);
float e= pow((1-swi),4);
kro[a]=d/e;
if (krw[a]==0)
```

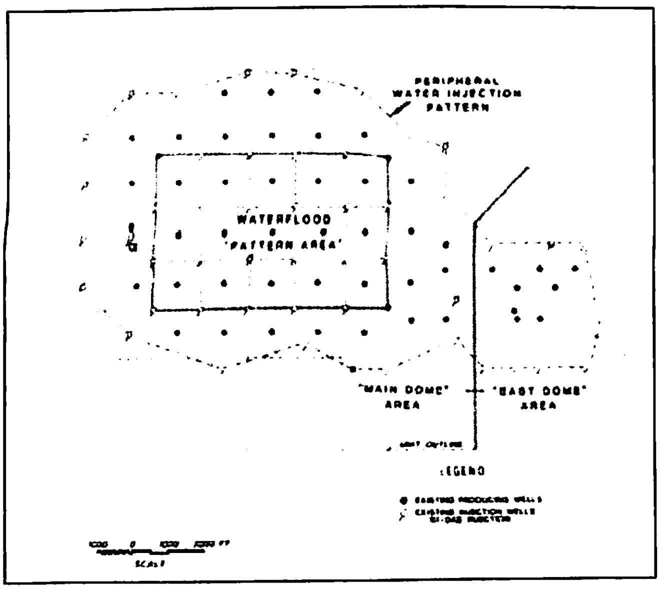
```
{fw[a]=0;}
if (krw[a]>0.0)
{
  m[a]=(visw*kro[a])/(krw[a]*viso); /* mobility ratio cal*/
   n[a]=m[a]+1;
  fw[a]=1/n[a];
}
}
 cout<<"Sw\t"<<"So\t"<<"  S*\t"<<"  S*w\t"<<"   Krw\t"<<" Kro\t"<<"  Fw\t"<<"\n";
 for( a=1;a<=10;a++)
 {
  cout<<sw[a]<<"\t"<<so[a]<<"\t";
  printf("%f ",s1[a]);
  cout<<" ";
  printf("%f ",sw1);
  cout<<" ";
  printf("%f ",krw[a]);
  cout<<" ";
  printf("%f ",kro[a]);
  cout<<" ";
   printf("%f ",fw[a]);
   cout<<"\n";
 }
 }
  float slope[50];
  for( b=1;b<=a-1;b++)
  {
    slope[b]= (fw[b+1]-fw[b])/(sw[b+1]-sw[b]);
  }
   int c;
   for(b=1;b<=a-6;b++)
  {
 if(slope[b]>slope[b+1])
  { c= b; }
 }
 float x1,y1;
 y1=(fw[c]+fw[c+1])/2;
 x1=(sw[c]+sw[c+1])/2;
 float line_slope= (y1-fw[1])/(x1-sw[1]);
 float j= -(line_slope*sw[1]);
 float sw_bt= (1-j)/line_slope;
 cout<< "\n\n swbt= "<< sw_bt;
 for (b=1;b<=a-1;b++)
   {
    dif1[b]=(fw[b+1]-fw[b]);
   }
```

```
 dif1[a]=0;
 dif2[a-1]=0;
 dif2[a]=0;
for (b=1;b<=a-2;b++)
{
  dif2[b]=dif1[b+1]-dif1[b];
}
float deri[50];
cout<<"\n\n"<<"Sw"<<"\t"<<"dsw/dfw";
for(b=1;b<=a-1;b++)
{
   deri[b]= (1/(sw[b+1]-sw[b]))*(dif1[b]-(dif2[b]/2));
   cout<<"\n"<<sw[b];
   cout<<"\t"<<deri[b];
}
cout<<"\n % w c "<<"\t Water sat. "<<"\t Sw-Swi "<<"\t (Swi-Swi)/Swi "<<"\t R.F
"<<"\n"<<" ";;
float sw_wc[25],rec[25],rf[25];
float sw_w=( sw[10]-sw_bt)/10;
// for cal. % water cut.
for(int s=0;s<=10;s++)
{ sw_wc[s]=sw_bt+(s*sw_w);
  rec[s]=(sw_wc[s]-swi)/(1-swi);
  rf[s]=rec[s]*0.65;
  cout<<s*10<<"\t ";
  printf("%f \t",sw_wc[s]);
  cout<<"  ";
  printf("%f \t",sw_wc[s]-swi);
  cout<<" ";
  printf("%f \t",rec[s]);
  cout<<" ";
  printf("%f \t",rf[s]);
  cout<<"\n";
 }
getche();
}
```

Appendix B-1

The West Seminole field shown produces from the San Andres formation in west Texas at an average depth of 5,100 ft [1555 m]. 37,38 is a cross section of the reservoir revealing a large dome on the west end and a smaller dome on the east end of the structure. A large gas cap covers most of the field, and there is a WOC near the bottom of the reservoir. The reservoir is geologically complex and heterogeneous, The reservoir was discovered in 1948 and was developed on approximately 40-acre [161 880-m2] well spacing. Reservoir pressure declined rapidly during primary production leading to fluid injection projects in an attempt to arrest pressure decline. Reinjection of produced gas began in 1964 with limited results. A peripheral waterflood was initiated in 1969-71 in the wells . Peripheral wells were completed at or below the WOC to avoid possible loss of oil into the gas cap, as depicted in Fig. 6.42. The peripheral water injection program, however, did not produce a significant change in reservoir pressure.

During 1973-75, fifteen 40-acre [161 880-m2] five spot patterns were developed in the main dome as shown in Fig



Data from new wells provided insight into the geology of the reservoir, several geological units that correlated across the entire field, suggesting that vertical barriers to flow might exist, were identified. These interpretations, combined with other data, led to a decision to use reservoir simulation to investigate the possibility of vertical movement of oil into the gas cap under the existing waterflooding program.

RESERVOIR ROCK AND FLUID PROPERTIES	
k. md	10
H, ft	50
Φ.fraction	0.15
Δp, psi	2,000
r_w, ft	0.75
μ_o, cp	2.75
μ_w. Cp	1.0
S_{iw},	0.25
S_{or},	0.30
K_{ro} at s_{iw}	0.7 (assumed)
K_{rw} at S_{or}	0.15 (assumed)
S_{gi}	0.10 (assumed)

Solutions

```
PROGRAM TO ESTIMATE 5 SPOT PATTERN EFF.
PROGRAM DESIGNED BY MANVENDRA SINGH ARYA & ROHIT MITRA,
B.TECH. (APE), UNIVERSITY OF PETROLEUM & ENERGY STUDIES.


 ENTER MENU....

Enter pattern area (acres) :                                40
Enter thickness (ft) :                                      50
Enter radius of injection to producing well (ft) :          .75
Enter siw (frac.) :                                         .25
Enter sor (frac.) :                                         .30
Enter porosity (frac.) :                                    .15
Enter permeability to liquid (darcies) :                    .10
Enter kro at siw (frac.) :                                   .7
Enter krw at sor (frac.) :                                  .15
Enter oil viscosity (cp) :                                  2.75
Enter water viscosity (cp) :                                 1
Enter initial gas saturation (frac.) :                      .10
Enter trapped gas saturation (frac.) :                      0.0
Enter pressure difference (psi.) :                          2000
```

Vol. of water injected at interference :	182802 bbl
Radius of flood front saturation :	311.14 ft

Mobility Ratio :	0.589286(Fraction)
The Injection Rate At Interference :	1641.62 Bbl/Day
Vol. Of Water Injected To Fill Up :	232734 Bbl
The Radius Of Water Front At Fill Up:	351.07
The Reservoir Radius :	526.604 Ft.
Max. Water Injection :	523651 Bbl
Injection Rate At Instant Fill Up :	1036.4 Bbl/Day
Injection Rate Between Interface And Fill Up :	150.216 Bbl/Day

Appendix B-2

Considering a an Example to check the validity of program

```
PROGRAM TO ESTIMATE 5 SPOT PATTERN EFF.
PROGRAM DESIGNED BY MANVENDRA SINGH ARYA & ROHIT MITRA,
B.TECH. (APE), UNIVERSITY OF PETROLEUM & ENERGY STUDIES.


 ENTER MENU....


Enter pattern area (acres) :5
Enter thikness (ft) :15
Enter radius of injection to producing well (ft) :0.5
Enter siw (frac.) :0.27
Enter sor (frac.) :0.3
Enter porosity (frac.) :.15
Enter permeability to liquid (darcies) :0.203
Enter kro at siw (frac.) :0.7
Enter krw at sor (frac.) :0.15
Enter oil viscosity (cp) :2
Enter water viscosity (cp) :1
Enter initial gas saturation (frac.) :0.10
Enter trapped gas saturation (frac.) :0.0
Enter pressure difference (psi.) :330

Vol. of water injected at interference : 6855.09 bbl
Radius of flood front saturation : 112.534 ft
mobility ration : 0.428571(fraction)
the injection rate at interference 186.326 bbl/day
vol. of water injected to fill up : 8727.52 bbl
 the radius of water front at fill up: 126.977
the reservoir radius : 186.183 ft.
max. water injection : 18764.2 bbl
injection rate at instant fill up : 129.572 bbl/day
Injection rate between interface and fill up : 134.961 bbl/day
```

Results conside with results inputed in text chap 6.0

Appendix B-3

Program listing written on C platform for the reference solution of appendix B-1 and B-2 (5 Spot pattern Efficiency).

✓	**ac**	**pattern area(acres)**
✓	**th**	**thickness(ft)**
✓	**rad**	**radius of injection to producing well(ft)**
✓	**siw**	**siw**
✓	**sor**	**sor**
✓	**por**	**porosity(frac.)**
✓	**per**	**permeability to liquid(darcies)**
✓	**kro_siw**	**Relative Permeability of oil at siw (frac.)**
✓	**krw_sor**	**Relative Permeability of water at sor (frac.)**
✓	**viso**	**oil viscosity(cp.)**
✓	**visw**	**water viscosity(cp.)**
✓	**sgi**	**initial gas saturation(frac.)**
✓	**sgt**	**trapped gas saturation(frac.)**
✓	**press**	**pressure difference(psi.)**
✓	**d**	**distance b/w injection and producing well.**
✓	**rob**	**rob is interference.**
✓	**wii**	**Vol. of water injected at interference**
✓	**rf**	**Radius of flood front saturation**
✓	**i**	**the injection rate at interference**
✓	**wif**	**vol. of water injected to fill up**
✓	**nw_rf**	**radius of water front at fill up**
✓	**re_ac**	**reservoir radius**
✓	**wi**	**max. water injection**
✓	**i2**	**injection rate at instant fill up**
✓	**i3**	**Injection rate between interface and fill up**

```
# include<stdio.h>
# include<iostream.h>
# include<conio.h>
# include<math.h>
void main()
{
 float ac,th,rad,siw,sgi,sgt,sor,por,per,kro_siw,krw_sor,viso,visw,d,rob,wii,press;
 int a,b;
 cout<<" PROGRAM TO ESTIMATE 5 SPOT PATTERN EFF. \n ";
 cout<<" PROGRAM DESIGNED BY MANVENDRA SINGH ARYA & ROHIT
MITRA, \n ";
 cout<<" BTECH. (APE), UNIVERSITY OF PETROLEUM & ENERGY STUDIES. \n
\n \n \n ";
 cout<<" ENTER MENU.... \n \n \n \n";
 cout<<" enter pattern area (acres) :";
```

```
cin>>ac;
cout<<" enter thickness (ft) :";
cin>>th;
cout<<" enter radius of injection to producing well (ft) :";
cin>>rad;
cout<<" enter siw (frac.) :";
cin>>siw;
cout<<" enter sor (frac.) :";
cin>>sor;
cout<<" enter porosity (frac.) :";
cin>>por;
cout<<" enter permeability to liquid (darcies) :";
cin>>per;
cout<<" enter kro at siw (frac.) :";
cin>>kro_siw;
cout<<" enter krw at sor (frac.) :";
cin>>krw_sor;
cout<<" enter oil viscosity (cp) :";
cin>>viso;
cout<<" enter water viscosity (cp) :";
cin>>visw;
cout<<" enter initial gas saturation (frac.) :";
cin>>sgi;
cout<<" enter trapped gas saturation (frac.) :";
cin>>sgt;
cout<<" enter pressure difference (psi.) :";
cin>>press;
float s=( ac * 43560)/2;
d= pow (s,.5); // d is distance b/w injection and producing well.
rob=d/ pow (2,.5); // rob is interference.
// vol.of water injected @ interference...
wii= (3.14159*d*d*por*th*(sgi-sgt))/2; // convert cu ft to bbl in next step
wii=wii*0.178108;
cout<<" \n Vol. of water injected at interference : "<< wii <<" bbl ";
        // radius of flood front...
        float rf; float sw=.70;
        float e =(wii*5.615)/(3.14159*por*th*(sw-siw));
        float f= e+ pow (rad,2);
        rf= pow(f,.5);
        cout<<" \n Radius of flood front saturation : "<< rf <<" ft ";
        float c= krw_sor/visw;
        float g= kro_siw/viso;
        float m= c/g;
        float h= 7.082*per*g*th*press;
        float l=((1/m)* log(rf/rad))+ log(rob/rf);
        float i=h/l; //cout<< d<< " "<<wii<<" "<<rf<<" "<<m<<" "<<h<<" "<<l;
```

```
        cout<<" \n ";
        cout<< "\n mobility ratio : "<< m<< "(fraction)";
        cout<<"\n";
        cout<< " the injection rate at interference "<< i <<" bbl/day ";

        float wif= 2*d*d*por*th*sgi;
        wif=wif/5.615; // to convert into bbl.
        cout<< "\n vol. of water injected to fill up : "<< wif<< " bbl ";
        float nw_rf;    // radius water front @ fill up.
        float j =(wif*5.615)/(3.14159*por*th*(sw-siw));
        float k= j+ pow (rad,2);
        nw_rf= pow(k,.5);       // nw_rf is in feets.
// for entire pattern to be
 cout<< "\n  the radius of water front at fill up: "<< nw_rf;
 float re_ac= d/(pow(3.14159,.5));
 cout<<" \n the reservoir radius : "<< re_ac <<" ft. ";
 float wi;
 wi= (3.14159*re_ac*re_ac*(sw-siw)*th*por)/5.615;
 cout<<"\n max. water injection : "<< wi <<" bbl ";
 float if_fill;
 float h2= 7.082*per*g*th*press;
 float l2=((1/m)* log(nw_rf/rad))+ log(re_ac/nw_rf)+ log(re_ac/rad);
 float i2=h2/l2;
 cout<<" \n injection rate at instant fill up : "<< i2 <<" bbl/day";
 float inj_fill;
 float l3=((1/m)*log(nw_rf/rad))+ log(nw_rf/rad);
 float i3= h2/l3;
 cout<<"\n Injection rate between interface and fill up : "<< i3<< " bbl/day ";
getche();
}
```

Appendix C-1

```
PROGRAM TO ESTIMATE AREAL SWEEP EFFICIENCY AT BREAKTHROUGH
AND FRACTION OF AREA SWEEPT TO SATURATION.
PROGRAM DESIGNED BY MITRA ROHIT & ARYA MANVENDRA SINGH,
B.TECH. (APE), UNIVERSITY OF PETROLEUM & ENERGY STUDIES.

ENTERING INPUT MENU...

 INPUT MENU...
 Enter choice for moblity ratio:
 1. Normal Mobility Ratio
 2. Shock Mobility Ratio

INPUT : 1

 Enter mobility ratio :0.43

 Enter cumulative water injected: 18768

 Enter cumulative water injected at break through: 75072

 Areal Sweep efficiency at breakthrough = 0.636271
 Fraction of area sweept to saturation = 0.255167
```

REMARK:
We have taken the calculated value of cumulative water injection from the program to "Estimate the 5 spot pattern efficiency" and assumed the cumulative water injection at break through as 75072 ie. 4 times cumulative water injected.

Appendix C-2

Program listing written on C platform for the reference solution of appendix D-1 (Areal sweep Efficiency calculations).

✓	**ms**	**shock mobility ratio**
✓	**m**	**mobility ratio**
✓	**wi**	**cummulative water injected**
✓	**wibt**	**cummulative water injected at break through**
✓	**eabt**	**areal sweep efficiency at breakthrough**
✓	**ea**	**Fraction of area swept to saturation**

```
# include <iostream.h>
# include <conio.h>
# include <math.h>
void main()
{

cout<<" PROGRAM TO ESTIMATE AREAL SWEEP EFFICIENCY AT
BREAKTHROUGH AND \n ";
 cout<<" FRACTION OF AREA SWEEPT TO SATURATION. \n";
cout<<" PROGRAM DESIGNED BY MITRA ROHIT & ARYA MANVENDRA
SINGH, \n ";
cout<<" B.TECH. (APE), UNIVERSITY OF PETROLEUM & ENERGY STUDIES. \n
";
cout<<" ENTERING INPUT MENU...\n ";

 float m,ms,eabt,ea,wi,wibt;
 int ch;
 cout<<" \n INPUT MENU...";
 cout<<" \n Enter choice for mobility ratio: ";
 cout<<" \n 1. Normal Mobility Ratio";
 cout<<" \n 2. Shock Mobility Ratio \n";
 cin>> ch;
 if (ch>=3)
 {
 cout<<" \n wrong choice run program again ";
  }
        if (ch==1)
         {
        cout<<" \n Enter mobiliy ratio :";
        cin>>m;
         if(m >= 1)
               {
               eabt= 0.794 + 0.1179 * log10(m-.5);
```

```
                }
        if (m < 1)
        {
         eabt= 0.515 + 0.7807 * log10(m+1);
        }
        }
if (ch==2)
 {
 cout<<" \n Enter shock mobility ratio :";
 cin>>ms;
 eabt= .54602036+(.3170817/ms)+(.30222997/(exp(ms))-(.00509693*ms));
  }
  if (ch<3)
  {
 cout<< " \n Enter cummulative water injected : ";
 cin>> wi;
 cout<< "\n Enter cummulative water injected at break through : ";
 cin>>wibt;
 ea= eabt + .633 * log10(wi/wibt);
 cout<< "\n Areal Sweep efficiency at breakthrough = "<<eabt;
 cout<< "\n Fraction of area swept to saturation = "<<ea; }
 getch();
 }
```

Appendix D-1

```
PROGRAM TO GENERATE THE FRACTIONAL FLOW CURVE
B/W SW V/S FW USING BUCKLET LAVETH AND CALCULATING THE
EFFIENCY
PROGRAM DESIGNED BY MITRA ROHIT & ARYA MANVENDRA SINGH,
B.TECH. (APE), UNIVERSITY OF PETROLEUM & ENERGY STUDIES.
ENTERING INPUT MENU...


  Enter oil viscosity: 1

  Enter water viscosity: .5

  Enter the number of saturation values that will be entered: 9


  1. Enter value of water saturation: .1

     Enter value of relative oil permeability: 1

     Enter value of relative water permeability: 0

  2. Enter value of water saturation: .3

     Enter value of relative oil permeability: .373

     Enter value of relative water permeability: .07

  3. Enter value of water saturation: .4

     Enter value of relative oil permeability: .21

     Enter value of relative water permeability: .169

  4. Enter value of water saturation: .45

     Enter value of relative oil permeability: .148

     Enter value of relative water permeability: .226

  5. Enter value of water saturation: .5

     Enter value of relative oil permeability: .1

     Enter value of relative water permeability: .3
```

```
6. Enter value of water saturation: .55

   Enter value of relative oil permeability: .061

   Enter value of relative water permeability: .376

7. Enter value of water saturation: .6

   Enter value of relative oil permeability: .033

   Enter value of relative water permeability: .476

8. Enter value of water saturation: .65

   Enter value of relative oil permeability: .012

   Enter value of relative water permeability: .6

9. Enter value of water saturation: .7

   Enter value of relative oil permeability: 0.0

   Enter value of relative water permeability: .74
```

s_no	sw	Kro	krw	mob	fw
1	0.1	1.0000004	0.0000004	0.0000004	0
2	0.3	0.3730004	0.0700004	2.6642864	0.272904
3	0.4	0.2100004	0.1690004	0.6213024	0.616788
4	0.45	0.1480004	0.2260004	0.3274344	0.753333
5	0.5	0.1000004	0.3000004	0.1666674	0.857143
6	0.55	0.0610004	0.3760004	0.0811174	0.924969
7	0.6	0.0330004	0.4760004	0.0346644	0.966497
8	0.65	0.0120004	0.6000004	0.0100004	0.990099
9	0.7	0.0000004	0.7400004	0.0000004	1

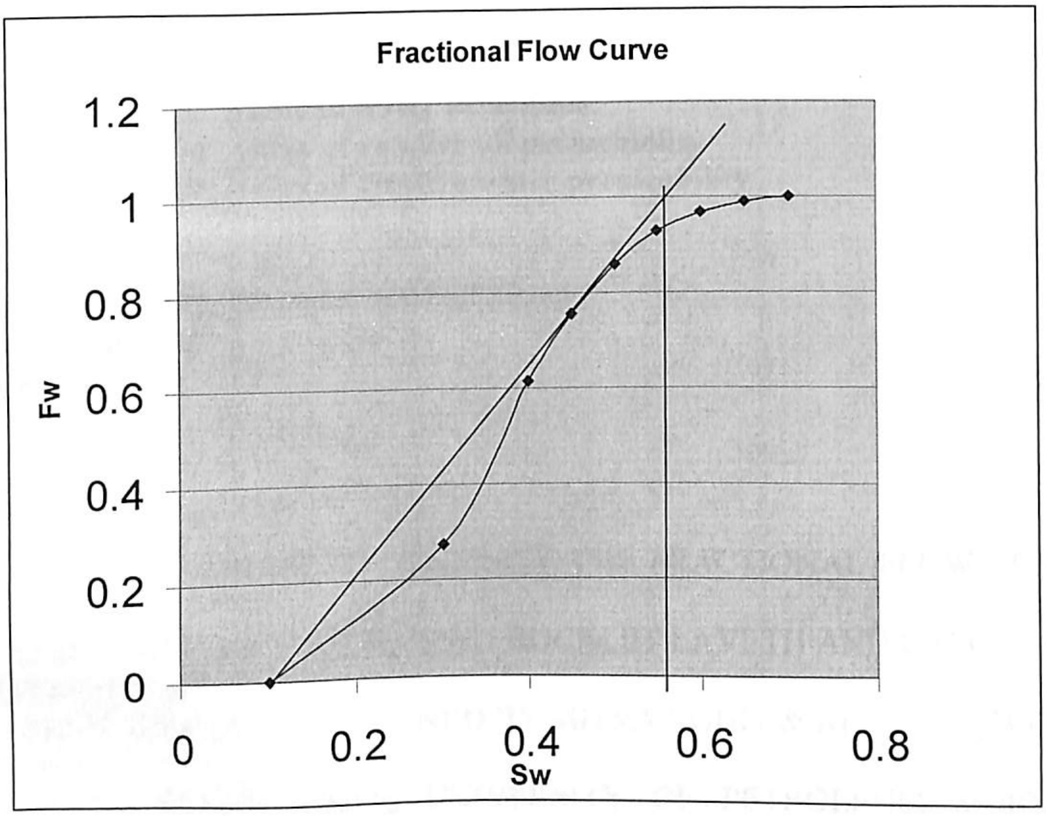
Water Break through **S_{wbt}= 0.636646**

Sw	dsw/dfw
0.1	1.18707
0.3	4.47553
0.4	3.05826
0.45	2.43602
0.5	1.61951
0.55	1.00983
0.6	0.609038
0.65	0.19802
0.7	0

% water cut	water satu	Sw-Swi	(Sw-Swi)/(1-Swi)	Recovery Factor
0	0.6366464	0.536646	0.5962744	0.3875784
10	0.6429824	0.542982	0.6033134	0.3921534
20	0.6493174	0.549317	0.6103524	0.3967294
30	0.6556524	0.555652	0.6173914	0.4013044
40	0.6619884	0.561988	0.6244314	0.4058804
50	0.6683234	0.568323	0.6314704	0.4104564
60	0.6746584	0.574658	0.6385094	0.4150314
70	0.6809944	0.580994	0.6455494	0.4196074
80	0.6873294	0.587329	0.6525884	0.4241824
90	0.6936654	0.593665	0.6596274	0.4287584
100	0.7000004	0.6	0.6666674	0.4333334

To check the validity we generated the Flow curve separately and checked the Breakthrough saturation

Generating Fractional Flow Curve Sw (x Axis) vs Fw (y Axis)



Both Break through Saturations consides with little Deviations

Appendix D-2

Program listing written on C platform for the reference solution of appendix E-1 (Fractional flow curve generation, Break Through and Recovery Calculations).
viso-oil viscosity

- ✓ **viso oil viscosity**
- ✓ **visw water viscosity**
- ✓ **sw[b] value of water saturation**
- ✓ **kro[b] value of relative oil permeability**
- ✓ **krw[b] value of relative water permeability**

```
// program to calculate the water flood efficiency.
# include <iostream.h>
# include <conio.h>
# include <math.h>
# include <stdio.h>
void main()
{
        cout<<" PROGRAM TO GENERATE THE FRACTIONAL FLOW CURVE \n
";
        cout<<" B/W SW V/S FW USING BUCKLET LAVETH AND CALCULATING
THE EFFIENCY \n";
        cout<<" PROGRAM DESIGNED BY MITRA ROHIT & ARYA MANVENDRA
SINGH, \n ";
        cout<<" BTECH. (APE), UNIVERSITY OF PETROLEUM & ENERGY
STUDIES. \n ";
        cout<<" ENTERING INPUT MENU...\n \n \n \n";
        int a,b;
        float dif1 [25],dif2[25];
float sw[25], kro[25], krw[25], fw[25],m[25],n[25];
float viso,visw;
cout<< "\n enter oil viscosity: ";
cin>>viso;
cout<<"\n enter water viscosity: ";
cin>>visw;
cout<<"\n enter the number of saturation values that will be entered: ";
cin>>a;
cout<<"\n \n";
for( b=1;b<=a;b++)
 {
        cout<< b<< ".";
        cout<<" Enter value of water saturation: ";
        cin>> sw[b];
        cout<<" \n Enter value of relative oil permeability: ";
```

```
        cin>> kro[b];
        cout<<" \n Enter value of relative water permeability: ";
        cin>> krw[b];
        if ( krw[b]==0) { fw[b]=0;}
        if (krw[b]>0.0)
        {
        m[b]=(visw*kro[b])/(krw[b]*viso); /* mobility ratio cal*/
        n[b]=m[b]+1;
        fw[b]=1/n[b];
        }
        cout<<" \n";
}
float slope[100];
cout<< " s_no "<< " sw "<< " kro "<< " krw "<< " mob "<< " fw ";
cout<<" \n \n ";
for( b=1;b<=a;b++)
{
        cout<< b<< ".";
        cout<<" ";
        printf("%f4", sw[b]);
        cout<<"\t";
        printf("%f4", kro[b]);
        cout<<"\t";
        printf("%f4",krw[b]);
        cout<<"\t";
        printf("%f4",m[b]);
        cout<<"\t";
        printf("%f",fw[b]);
        cout<<" \n ";
}

for( b=1;b<=a-1;b++)
        {
         slope[b]= (fw[b+1]-fw[b])/(sw[b+1]-sw[b]);
        }
          int c;
          for(b=1;b<=a-2;b++)
                {
                if(slope[b]>slope[b+1])
                            { c= b; }
                }
                float x1,y1;
        y1=(fw[c]+fw[c+1])/2;
        x1=(sw[c]+sw[c+1])/2;
        float line_slope= (y1-fw[1])/(x1-sw[1]);
        float j= -(line_slope*sw[1]);
```

```
        float sw_bt= (1-j)/line_slope;
        cout<< "\n\n swbt= "<< sw_bt;
        for (b=1;b<=a-1;b++)
                {
                 dif1[b]=(fw[b+1]-fw[b]);
                }
         dif1[a]=0;
         dif2[a-1]=0;
         dif2[a]=0;
        for (b=1;b<=a-2;b++)
        {
         dif2[b]=dif1[b+1]-dif1[b];
        }
        float deri[25];
        cout<<"\n\n"<<"Sw"<<"\t"<<"dsw/dfw";
        for(b=1;b<=a;b++)
        {
         deri[b]= (1/(sw[b+1]-sw[b]))*(dif1[b]-(dif2[b]/2));
         cout<<"\n"<<sw[b];
         cout<<"\t"<<deri[b];
        }
        cout<<"\n % water cut "<<" Water sat. "<<" Sw-Swi "<<" (Swi-Swi)/Swi "<<"
    R.F "<<"\n";
        float sw_wc[100], rec[100],rf[100];
        float sw_w= ( sw[a]-sw_bt)/10;
        // for cal. % water cut.
        for(int s=0;s<=10;s++)
        { sw_wc[s]=sw_bt+(s*sw_w);
         rec[s]=(sw_wc[s]-sw[1])/(1-sw[1]);
         rf[s]=rec[s]*0.65;
         cout<<s*10<<" ";
         printf("%f4",sw_wc[s]);
         cout<<"  ";
         printf("%f4",sw_wc[s]-sw[1]);
         cout<<" ";
         printf("%f4",rec[s]);
         cout<<" ";
         printf("%f4",rf[s]);
         cout<<"\n";
        }
        getche();
    }
```