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| **Name:**<br><br>**Enrolment No:** | UPES<br>UNIVERSITY WITH A PURPOSE |

**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**
**End Semester Examination, December 2019**

**Course: Logic and Sets** **Semester: III**
**Program: B.Sc. (Hons.) Mathematics** **Time 03 hrs.**
**Course Code: MATH 2032** **Max. Marks: 100**

**Instructions: All questions are compulsory. Question number 11 in section C has internal choice.**

**SECTION A**

| S. No. | | Marks | CO |
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| Q 1 | By using Venn diagram, show that the following argument is valid:<br>S1: All my tin objects are saucepans.<br>S2: I find all your presents very useful.<br>S3: None of my saucepans is of the slightest use.<br>―――――――――――――――――<br>S: Your presents to me are not made of tin. | **5** | **CO4** |
| Q 2 | Test the validity of following argument:<br>If two sides of a triangle are equal, then the opposite sides are equal.<br>Two sides of a triangle are not equal<br>―――――――――――――――――<br>The opposite angles are not equal | **5** | **CO2** |
| Q 3 | Let $A, B, C$ be arbitrary sets. Show that $(A-B)-C=(A-C)-(B-C)$ | **5** | **CO3** |
| Q 4 | The relation $R$ on a set $A=\{1,2,3,4\}$ is defined by $R=\{(1,3),(1,4),(3,2),(3,3),(3,4)\}$ draw the directed graph of $R$ and hence $R^{-1}$. | **5** | **CO5** |

**SECTION B**

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| Q 5 | Consider the following assumptions:<br>S1: All dictionaries are useful. S2: Mary owns only thriller novels. S3: No thriller novel is useful.<br>Use a Venn diagram to determine the validity of each of the following conclusions:<br>i. Thriller novels are not dictionaries.<br>ii. Mary does not own a dictionary.<br>iii. All useful books are dictionaries. | **10** | **CO4** |
| Q 6(a) | If $P(n,k)=P(n-1,k-1)+kP(n-1,k)$ with $P(n,1)=P(n,n)=1$, where $P(n,k)$ is the number of partitions of a set with $n$ elements into $k$ subsets then find $P(4,2)$. Compare your result by listing a partition of $\{x,y,z,w\}$. | **04** | **CO3** |

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| Q 6(b) | Using De Morgan's laws prove that $(A \cup B)\backslash(A \cap B) = (A\backslash B) \cup (B\backslash A)$ | **06** | |
| Q 7 | Let $R$ and $S$ be relation from $A$ to $B$, show that<br>i. $if\ R \subseteq S, then\ R^{-1} \subseteq S^{-1}$<br>ii. $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$<br>iii. $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$ | **10** | **CO5** |
| Q 8 | Consider the following five relations:<br>(1) Relation $\leq$ (less than or equal) on the set $Z$ of integers.<br>(2) Set inclusion $\subseteq$ on a collection $C$ of sets.<br>(3) Relation $\perp$ (perpendicular) on the set $L$ of lines in the plane.<br>(4) Relation $\parallel$ (parallel) on the set $L$ of lines in the plane.<br>(5) Relation \| of divisibility on the set $N$ of positive integers. (Recall x\|y if there exists $z$ such that $xz = y$<br>Determine which of the relations are reflexive**.** | **10** | **CO5** |
| Q 9<br><br><br><br><br><br><br><br><br><br><br><br>Q 9(a) | Solve this Zebra puzzle. Five men with different nationalities and with different jobs live in consecutive houses on a street. These houses are painted in different colors. The men have different pets and have different favorite drinks. Determine who owns a zebra and whose favorite drink is mineral water (which is one of the favorite drink) given these clues:<br>The Englishman lives in the red house. The Spaniard owns a dog. The Japanese man is a painter. The Italian drinks tea. The Norwegian lives in the first house on the left. The green house is immediately to the right of the white one. The photographer breeds snails. The diplomat lives in the yellow house. Milk is drunk in the middle house. The owner of green house drinks coffee. The Norwegian's house is next to the blue one. The violinist drinks orange juice. The fox is in a house next to that of the physician. The horse is in a house next to that of the diplomat.<br>**OR**<br>You live on island where there are only two kinds of people: the ones who always tell the truth (truth tellers) and those who always lie (liars). You are accused of crime and brought before the court, where you are allowed to speak only one sentence in your defense. What do you say in each of the following situations?<br>i. If you were a liar, (the court does not know that) and you were innocent. and it is an established fact that a liar committed the crime.<br>ii. Same situation as above, but you are the one who committed the crime.<br>iii. If you were a truth teller, (the court does not know that) and you were innocent. In addition, it is an established fact that a truth teller committed the crime.<br>iv. If you were innocent and it is an established fact that the crime was not committed by a "normal" person. Normal people are that new immigrant | **20**<br><br><br><br><br><br><br><br><br><br><br><br>**10** | **CO1** |

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| Q 9(b) | group who sometimes lie and sometimes speak the truth. What sentence, no matter whether you were a truth teller, liar or normal, can you prove your innocence?<br><br>Determine whether the following propositions are a tautology, contingency or contradiction :<br>$(i)\ p \to (p \to q)$ $(ii)\ p \to (q \to p)$ $(iii)\ (p \to q) \wedge (p \wedge r) \to q.$ | **10** | |
| Q10(a) | In a survey of 500 students of a college, it was found that 49% liked watching football, 53% liked watching hockey and 62% liked watching basketball. Also, 27% liked watching football and hockey both, 29% liked watching basketball and hockey both, 5% liked watching none of these games. Draw a Venn diagram to represent the following:<br>i. How many students like watching all the three games<br>ii. Find the ratio of number of students who like watching only football to those who like watching hockey.<br>iii. Find the number of students who like watching only one of the three games<br>iv. Find the number of students who like watching at least two of the given games. | **10** | **CO4** |
| Q10(b) | Let $R$ be the relation on $N$ defined by $x + 3y = 12$, i.e. $R = \{(x, y) \mid x + 3y = 12\}$<br>i. Write $R$ as a set of ordered pairs.<br>ii. Find the domain and range of $R$<br>iii. Find $R^{-1}$<br>iv. Find the composition relation $RoR$ | **10** | **CO5** |