Name:

Enrolment No:

UPES
UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2019

Course: Mathematics III (Probability and statistics) **Semester: III**
Program: B. Tech. (ECE, Electrical) **Time 03 hrs.**
Course Code: MATH 2005 **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A

S. No.		Marks	CO
Q 1	Show that $\mathrm{Cov}(x,x) = \mathrm{Var}(x)$.	**4**	**CO4**
Q 2	If a linear relation exists between the variables x and y, then show that the coefficient of correlation between them is 1 or -1.	**4**	**CO4**
Q 3	There are 3 true coins and 1 false coin with head on both sides. A coin is chosen at random and tossed 4 times. If head occurs all the 4 times, what is the probability that the false coin has been chosen and used?	**4**	**CO1**
Q 4	Write down relation between correlation and regression coefficients.	**4**	**CO4**
Q 5	If on the average, 2 cars enter a certain parking lot per minute, what is the probability that during any given minute 4 or more cars will enter the lot?	**4**	**CO3**

SECTION B

Q 6	If $X_1, X_2, \ldots, X_n$ constitute a random sample from an infinite population with the mean μ and the variance σ^2, then show that $E(\bar{X}) = \mu$ and $var(\bar{X}) = {\sigma^2}/{n}$.	**10**	**CO4**
Q 7	If the probability density function of a continuous random variable is given by $$f = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ 3a - ax, & 2 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}.$$ i) Find the value of a. ii) If x_1, x_2 and x_3 are three independent observations of X, what is the probability that exactly one of these three is greater than 1.5?	**10**	**CO3**
Q 8	If X has the exponential distribution given by $$f(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}.$$ Find the probability density of the random variable $Y = \sqrt{X}$.	**10**	**CO3**

<table>
<tr>
<td>Q 9</td>
<td>Ten competitors in a beauty contest were ranked by three judges in the following orders:

First Judge : 1 6 5 10 3 2 4 9 7 8

Second Judge: 3 5 8 4 7 10 2 1 6 9

Third Judge : 6 4 9 8 1 2 3 10 5 7

Use the method of rank correlation to determine which pair of judges has the nearest approach to common taste in beauty?

OR

Ten students got the following percentage of marks in Economics and Statistics:

Roll No.: 1 2 3 4 5 6 7 8 9 10

Marks in Eco.: 78 36 98 25 75 82 90 62 65 39

Marks in Stats.:84 51 91 60 68 62 86 58 53 47

Calculate the coefficient of correlation.</td>
<td>10</td>
<td>CO1</td>
</tr>
<tr>
<td colspan="4">SECTION-C</td>
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<tr>
<td>Q 10</td>
<td>If the probability density of X is given by

$$f(x)=\begin{cases}\frac{1}{\theta}e^{-\frac{x}{\theta}}, & \text{for } x>0\\ 0 & \text{otherwise}\end{cases}$$

Find the mean and variance of X .</td>
<td>20</td>
<td>CO2</td>
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<tr>
<td>Q 11</td>
<td>The demand for a particular space part in a factory was found to vary from day to day. In a sample study, the following information was obtained:
<table>
<tr><th>Days</th><th>Mon</th><th>Tue</th><th>Wed</th><th>Thu</th><th>Fri</th><th>Sat</th></tr>
<tr><td>No. of parts demanded</td><td>1124</td><td>1125</td><td>1110</td><td>1120</td><td>1125</td><td>1116</td></tr>
</table>
Use chi-square to test the hypothesis that number of parts demanded does not depend on the day of the week at 5% level of significance. (Given $\chi^2_{5,0.05}=11.07$)</td>
<td>20</td>
<td>CO4</td>
</tr>
</table>

	OR A survey of 800 families having four children is as follows: No. of male births: 0 1 2 3 4 No. of female births: 4 3 2 1 0 No. of families: 32 178 290 236 64 Test whether the data are consistent with the hypothesis that the binomial law holds and the chance of male birth is equal to that of female birth. ($\chi^2_{4,\,0.05} = 9.49$)		