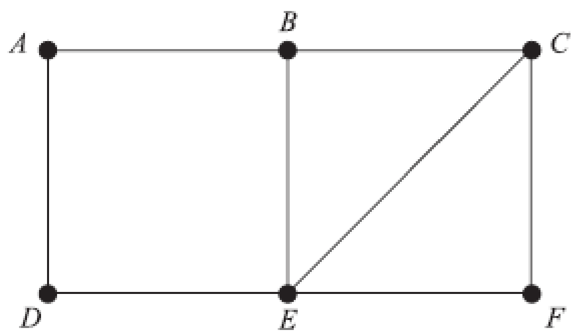
Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2019

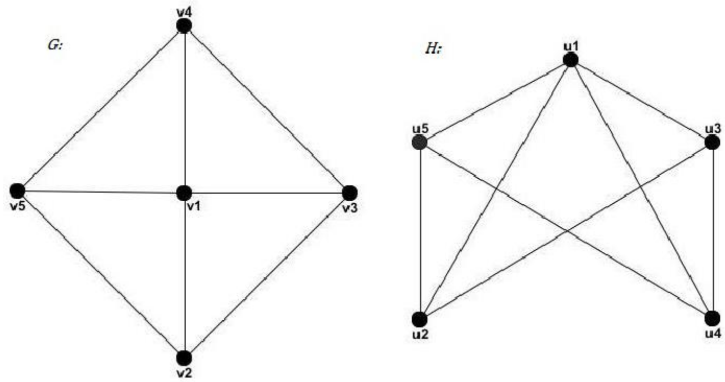
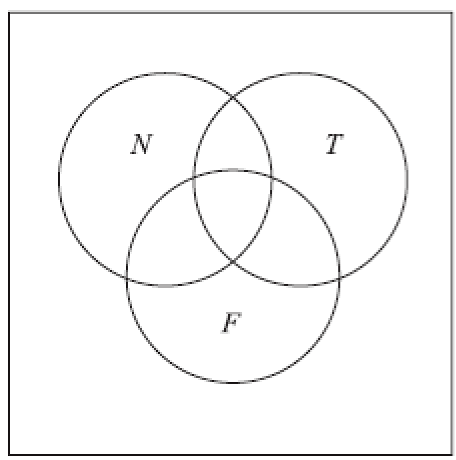
Program: B.Tech Civil Engineering with spl. in infrastructure development. **Semester: III**
Course: MATHEMATICS - 3 **Time 03 hrs.**
Course Code: MATH 2004 **Max. Marks: 100**

SECTION A
(Answer all the questions)

S. No.		**Marks**	**CO**
Q 1.	Find $L\{F(t)\}$ where $F(t) = \begin{cases} \sin(t - \frac{2\pi}{3}), & t > \frac{2\pi}{3} \\ 0, & t < \frac{2\pi}{3} \end{cases}$	**4**	**CO1**
Q 2.	Let $A, B \subseteq R^2$ where $A = \{(x,y): y = 2x + 1\}$, $B = \{(x,y): y = 3x\}$, $C = \{(x,y): y = x - 7\}$. Determine (i) $A \cap B$ (ii) $B \cap C$.	**4**	**CO2**
Q 3.	Consider the "division" relation of $S = \{1,2,3,4,6,9\}$. Draw the Hasse diagram.	**4**	**CO2**
Q 4.	Prove that in a group G, inverse of any element is unique.	**4**	**CO3**
Q 5.	Consider the following graph:  Find (a) All simple paths from A to F (b) All trails from A to F.	**4**	**CO4**

SECTION B
(Answer all the questions. Q 9 has internal choice)

Q 6.	Let $Z(u_n) = U(z)$, show that $Z(a^{-n}u_n) = U(az)$. Also prove that (i) $Z(\cos n\theta) = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1)}$ (ii) $Z(\sin n\theta) = \frac{z sin\theta}{z^2-2z\cos\theta+1)}$.	**10**	**CO1**

Q 7.	Define isomorphic graphs. Find whether the two graphs G and H given below are isomorphic or not.  	**10**	**CO4**
Q 8.	In a survey of 120 people, it was found that 65 read $Newsweek$ magazine, 45 read $Time$, 42 read $Fortune$, 20 read both $Newsweek$ and $Time$, 25 read both $Newsweek$ and $Fortune$, 15 read both $Time$ and $Fortune$, 8 read all three of them. (a) Find the number of people who read at least one of the three magazines. (b) Fill in the correct number of people in each of the eight regions of the Venn diagram given below where N, T and F denote the set of people who read $Newsweek$, $Time$ and $Fortune$ respectively.  (c) Find the number of people who read exactly one magazine.	**10**	**CO2**
Q 9.	Let G be a group. If $a, b \in G$ such that $a^4 = e$, the identity element of G and $ab = ba^2$, prove that $a = e$. **(OR)** Let Q be the set of positive rational numbers which can be expressed in the form $2^a 3^b$, where a and b are integers. Prove that the algebraic structure $(Q, ^\circ)$ is a group where $^\circ$ is multiplication operator.	**10**	**CO3**

SECTION-C
(Answer all the questions. Q 11A-Q 11B have internal choice)

Q 10A.	Solve $y''' + 2y'' - y' - 2y = 0, y(0) = y'(0) = 0$ and $y''(0) = 6$ using Laplace transform.	**10**	**CO1**

Q 10B.	Find the finite Fourier cosine transform of $F(x) = \left(1 - \frac{x}{\pi}\right)^2$.	**10**	**CO1**
Q 11A.	Prove that the set $\{0,1,2,3,4\}$ is a finite abelian group of order 5 under addition modulo 5 as composition. **(OR)** If a, b are arbitrary elements of a group G, show that $(ab)^2 = a^2b^2$ if and only if G is abelian.	**10**	**CO3**
Q 11B.	If H_1 and H_2 are two subgroups of a group G, then prove that $H_1 \cap H_2$ is also a subgroup of G. **(OR)** Prove that the order of each subgroup of a finite group G is a divisor of the order of the group G.	**10**	**CO3**