Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2019

Course: Mathematics III **Semester: III**
Program: B.Tech ASE & ASE+AVE **Time 03 hrs.**
Course Code: MATH 2009 **Max. Marks: 100**

Instructions: All questions are compulsory. Question number 11 in section C has internal choice.

SECTION A

S. No.		Marks	CO
Q 1	With usual notations, Prove that $\Delta^n\left(\frac{1}{x}\right) = (-1)^n \frac{n!h^n}{x\,(x+h)\ldots\ldots\ldots(x+nh)}$	**4**	**CO3**
Q 2	Use Simpson's 3/8 rule to evaluate $\int_0^{\pi} \frac{sin^2x}{(5+4cosx)}dx$, if $f(0) = 0, f\left(\frac{\pi}{6}\right) = 0.02954$, $f\left(\frac{\pi}{3}\right) = 0.10714,\ f\left(\frac{\pi}{2}\right) = 0.2,\ f\left(\frac{2\pi}{3}\right) = 0.25,\ f\left(\frac{5\pi}{6}\right) = 0.16277, f(\pi) = 0$.	**4**	**CO4**
Q 3	Express $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial notation. Hence, find $\Delta^3 f(x)$.	**4**	**CO3**
Q 4	Use Lagrange's method of interpolation to find the value of y when $x = 10$ from the following table: x: 5, 6, 9, 11 y: 12, 13, 14, 16	**4**	**CO4**
Q 5	Apply Newton's formula to find the value of $(30)^{1/5}$.	**4**	**CO2**

SECTION B

S. No.		Marks	CO
Q 6	Using Euler's method, find the approximate value of y at $x = 1.5$ taking $h = 0.1$ given $\frac{dy}{dx} = \frac{y-x}{\sqrt{xy}}$ $and\ y(1) = 2$.	**10**	**CO4**
Q 8	By using Bi-section method, find a real root between $0\ and\ 1$ of the equation $e^{-x} - x = 0$, correct up to three decimal places.	**10**	**CO2**
Q 9	A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of time t (seconds). Calculate the angular velocity and angular acceleration of the rod at $t = 0.6$ second. x: 0, 0.2, 0.4, 0.6, 0.8, 1.0 θ: 0, 0.12, 0.49, 1.12, 2.02, 3.20	**10**	**CO3**
Q 10	By means of Newton's divided difference formula, find the values of $f(8) and\ f(15)$ from the following table: x: 4, 5, 7, 10, 11, 13 $f(x)$: 48, 100, 294, 900, 1210, 2028	**10**	**CO3**

SECTION-C			
Q11(a)	A tightly stretched string with fixed end points $x = 0\ and\ x = l$ is initially in the equilibrium position. It is set vibrating by giving to each of its points Q velocity of $v_0 sin^3 \frac{\pi x}{l}$. Find the displacement $y(x, t)$.	10	CO1
(b)	If $F(D, D')z = f(x.y)$ is a linear homogeneous partial differential equation, where $F(D, D')$ is a homogeneous function of D and D' of degree n, then prove that the particular integral of the equation will be $z = \frac{1}{(D-mD')} f(x, y) = \int f(x, c - mx)dx$	10	
	OR		
Q11(a)	A laterally insulted bar of length l has its ends A and B maintained at 0℃ and 100℃ respectively until steady state conditions prevail. If the temperature at B is suddenly reduced to 0℃ and kept so while that of A is maintained at 0℃ , find the temperature at a distance x from A at any time t.	10	
(b)	Find the solution of $(D^3 - 7DD'^2 - 6D'^3)z = \mathrm{x}^2 + \mathrm{xy}^2 + \mathrm{y}^3 + \cos(\mathrm{x} - \mathrm{y})$	10	
Q12(a)	Apply the Runge-Kutta method of fourth order to find an approximate value of $y\ at\ x = 0.2$ if $\frac{dy}{dx} = x + y^2$, given that $y = 1\ when\ x = 0$ in steps of $h = 0.1$.	10	CO4
Q12(b)	Use Crout's method to solve following system of equations: $$x + 2y + z = 4$$ $$2x - 3y - z = -3$$ $$3x + y + 2z = 3$$	10	CO4