Name: **Enrolment No:**	UPES

End Semester Examination, December 2017

Course: MATH 1001– Mathematics I

Programme: B.Tech.- All SoE Branches
Semester: I (ODD-2017-18)
Time: 03 hrs. **Max. Marks:100**

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

Section A
(Attempt all questions)

1.	Find the eigen values of the following matrix. $A=\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$	**[4]**	**CO1**
2.	Examine whether the following set of vectors is linearly independent. $(1,2,3,4),\ (2,0,1,-2),\ (3,2,4,2).$	**[4]**	**CO1**
3.	Using double integration, find the area of the region bounded by the curve $xy = 16$ and the lines $y=x, y=0$ and $x=8.$	**[4]**	**CO3**
4.	If $u=u(y-z,\ z-x,\ x-y),$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0.$	**[4]**	**CO2**
5.	If x increases at the rate of $2\,cm/sec$ at the instant when $x=3cm$ and $y=1cm,$ at what rate must y be changing in order that the function $2xy-3x^2y$ shall be neither increasing nor decreasing?	**[4]**	**CO2**

SECTION B
(Q6-Q10 are compulsory and Q10 has internal choice)

6.	Using concept of curve tracing, draw the sketch of the curve $y^2(a-x)=x^2(a+x),\ a>0.$	**[8]**	**CO2**
7.	Find by triple integration, the volume of the paraboloid of revolution $x^2+y^2=4z$ cut off by the plane $z=4.$	**[8]**	**CO3**
8.	If $y^{\frac{1}{m}}+y^{-\frac{1}{m}}=2x,$ prove that $(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0.$	**[8]**	**CO2**
9.	If $f(x,y)$ and $\phi(x,y)$ are homogeneous function of x,y of degree p and q respectively and $u=f(x,y)+\phi(x,y),$ show that $f(x,y)=\frac{1}{p(p-q)}\left[x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}\right]-\frac{(q-1)}{p(p-q)}\left[x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right].$	**[8]**	**CO2**

10.	If $A=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$, then show that $A^n=A^{n-2}+A^2-I$ for $n\geq 3$. Hence find A^{100}. **OR** Find the matrix P that transforms the matrix $A=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$ to the diagonal form. Hence evaluate A^8.	**[8]**	**CO1**
SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice)**			
11(A).	Evaluate the following integral by changing to polar co-ordinates: $\int_0^2\int_0^{\sqrt{2x-x^2}}\frac{x}{x^2+y^2}\,dydx$	**[10]**	**CO3**
11(B).	Expand the function $f(x)=x\sin x$ as a Fourier series in the interval $-\pi\leq x\leq\pi$.	**[10]**	**CO4**
12(A).	Apply Dirichlet's theorem to find the volume of the solid surrounded by the surface $$\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}+\left(\frac{z}{c}\right)^{\frac{2}{3}}=1.$$ **OR** Let $\beta(p,q)$ represents beta function for $p,q>0$, then show that $\beta(p,q)=\int_0^\infty\frac{y^{q-1}}{(1+y)^{p+q}}\,dy=\int_0^1\frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}\,dx$	**[10]**	**CO3**
12(B).	Find the Fourier series corresponding to the function $f(x)$ defined in $(-2,2)$ as follows: $f(x)=\begin{cases}2 & in \quad -2<x\leq 0\\ x & in \quad 0<x<2.\end{cases}$ **OR** Find the half-range sine series of the function $f(x)=e^{ax}\quad for\quad 0<x<\pi.$	**[10]**	**CO4**

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Section A

(Attempt all questions)

1.	Find the value of k, for which the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & k \end{bmatrix}$ is at most 2.	**[4]**	**CO1**
2.	Using Cayley Hamilton theorem find A^8, where $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.	**[4]**	**CO1**
3.	Compute $\frac{\Gamma\left(-\frac{5}{2}\right)}{\Gamma\left(\frac{5}{2}\right)}$.	**[4]**	**CO3**
4.	Determine the following functions u, v and w, are functionally dependent or not. If functionally dependent, find the relation between them. (Note that, $\cosh^2\theta - \sinh^2\theta = 1$) $u = x^2 e^{-y}\cosh z\,, v = x^2 e^{-y}\sinh z\,, w = 3x^4 e^{-2y}$.	**[4]**	**CO2**
5.	If $z = xy\, f\left(\frac{y}{x}\right)$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$. Also, show that if z is a constant, then $\frac{f'\left(\frac{y}{x}\right)}{f\left(\frac{y}{x}\right)} = \frac{x\left\{y + x\frac{dy}{dx}\right\}}{y\left\{y - x\frac{dy}{dx}\right\}}$.	**[4]**	**CO2**

SECTION B

(Q6-Q9 are compulsory and Q10 has internal choice)

6.	Trace the curve $y^2(2 - x) = x^3$.	**[8]**	**CO2**
7.	Evaluate the integral: $$\int_0^1 \int_y^{y^{\frac{1}{3}}} e^{x^2}\, dxdy$$	**[8]**	**CO3**
8.	Evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$, where $$u + v + w = x + y + z$$ $$uv + vw + wu = x^2 + y^2 + z^2$$ $$uvw = \frac{1}{3}(x^3 + y^3 + z^3).$$	**[8]**	**CO2**

9.	Find the asymptotes of the curve $y = \frac{3x}{2}\log_e\left(e - \frac{1}{3x}\right)$.	**[8]**	**CO2**
10.	Diagonalize the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$. **OR** Examine whether the vectors $v_1 = (1,1,1,1), v_2 = (0,1,1,1), v_3 = (0,0,1,1)$ and $v_4 = (0,0,0,1)$ are linearly dependent or not. If dependent, find the relation between them.	**[8]**	**CO1**
SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice)**			
11.A	Using Beta-Gamma function evaluate $\int_0^1 t^2(1-t^4)^{-\frac{1}{2}}\,dt \times \int_0^1 (1+t^4)^{-\frac{1}{2}}\,dt$.	**[10]**	**CO3**
11.B	Expand $f(x)$ in Fourier series on $-\pi < x < \pi$ if $f(x) = \begin{cases} 0, & for -\pi < x < 0 \\ \pi, & for\ 0 < x < \pi. \end{cases}$	**[10]**	**CO4**
12.A	Find the volume of the solid under the surface $bz = x^2 + y^2$, and whose base R is the circle $x^2 + y^2 = b^2$. **OR** Find the area of the region enclosed by the curve $\left(\frac{x}{a}\right)^\alpha + \left(\frac{y}{b}\right)^\beta = 1$ in first quadrant, where $\alpha, \beta > 0$.	**[10]**	**CO3**
12.B	Find a Fourier series of $f(x) = x$ on the closed interval $0 \le x \le \pi$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$. **OR** Find the Fourier series of $f(x)$ on the closed interval $-1 < x \le 1$, where $f(x) = \begin{cases} x, for -1 < x \le 0 \\ x+2, for\ 0 < x \le 1. \end{cases}$ Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$.	**[10]**	**CO4**