Name: **Enrolment No:**	UPES

End Semester Examination, December 2017
Course: MATH-201-Mathematics-III
Programme: B. Tech (GSE, MINING, ASE, FSE, APEUP, Electronics, EE Broad band, EE IOT, GIE, ASE AVE, Electrical, PSE, Cyber Law)
Semester: III (ODD-2017-18)
Time: 03 hrs. **Max. Marks:100**

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks); attempt all questions from **Section C** (each carrying 20 marks).

	Section A **(Attempt all questions)**		
1.	Expand $\frac{1}{z^2-3z+2}$ for $1<\|z\|<2$.	**[4]**	**CO3**
2.	Evaluate $\int_C \frac{e^{2z}}{z^2+1}dz$, where C is $\|z\|=\frac{1}{2}$.	**[4]**	**CO3**
3.	Express the polynomial $f(x)=4x^3-2x^2-3x+8$ in terms of Legendre polynomials.	**[4]**	**CO2**
4.	Find the bilinear transformation which maps the points $z=0,-i,-1$ into $w=i,1,0$ respectively.	**[4]**	**CO4**
5.	The only singularities of a single valued function $f(z)$ are poles of order 1 and 2 at $z=-1$ and at $z=2$, with residues at these poles 1 and 2 respectively. If $f(0)=\frac{7}{4}$ and $f(1)=\frac{5}{2}$, determine the function.	**[4]**	**CO4**
	SECTION B **(Q6-Q9 are compulsory and Q10 has internal choice)**		
6.	Find the value of a and b such that the function $f(z)=x^2+ay^2-2xy+i(bx^2-y^2+2xy)$ is analytic. Also find $f'(z)$.	**[8]**	**CO3**
7.	Evaluate $\int_C \frac{\sin^6 z}{(z-\frac{\pi}{6})^3}dz$, where C is $\|z\|=1$.	**[8]**	**CO3**
8.	Solve the difference equation $y_{n+2}-2y_{n+1}+y_n=2^n$ by the generating function method with initial conditions $y_0=2$ and $y_1=1$.	**[8]**	**CO1**
9.	Prove the Rodrigues formula $P_n(x)=\frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$.	**[8]**	**CO2**

10.	Solve $\left(D^2 - DD' - 2D'^2\right)z = (y-1)e^x$. **OR** Solve $\left(D^2 + DD' - 6D'^2\right)z = x^2 \sin(x+y)$.	**[8]**	**CO5**
SECTION C **(Q11 is compulsory and Q12 have internal choice)**			
11.A	Apply the calculus of residues to evaluate the integral $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$.	**[10]**	**CO4**
11.B	Evaluate $\int_0^{2\pi} \frac{\cos 3\theta \, d\theta}{5 - 4\cos\theta}$.	**[10]**	**CO4**
12.	A tightly stretched string with fixed end points $x = 0$ and $x = \pi$ is initially in a position given by $y = x$, $0 < x < \pi$. If it is released from rest from this position, find the displacement $y(x,t)$. **OR** The ends A and B of a rod 20 cm long have the temperature at 30℃ and 80℃ until steady state prevails. The temperature of the ends are changed to 40℃ and 60℃ respectively. Find the temperature distribution in the rod at time t.	**[20]**	**CO5**

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Section A
(Attempt all questions)

1.	Expand $\frac{z^2-1}{(z+2)(z+3)}$ for $2 < \|z\| < 3$.	**[4]**	**CO3**
2.	Evaluate $\int_C \tan z\, dz$, where C is $\|z\| = 1$.	**[4]**	**CO3**
3.	Show that $\int_{-1}^{1} P_n(x)dx = 0,$ for $n \neq 0$.	**[4]**	**CO2**
4.	Show that the transformation $w = \frac{5-4z}{4z-2}$ transforms the circle $\|z\| = 1$ into a circle of radius unity in w-plane.	**[4]**	**CO4**
5.	The only singularities of a single valued function $f(z)$ are poles of order 1 and 2 at $z = -1$ and at $z = 2$, with residues at these poles 1 and 2 respectively. If $f(0) = \frac{7}{4}$ and $f(1) = \frac{5}{2}$, determine the function.	**[4]**	**CO4**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6.	Prove that $f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ satisfies Cauchy Riemann equations at the origin but $f'(0)$ does not exit.	**[8]**	**CO3**
7.	Evaluate $\int_C \frac{z+1}{z^4-4z^3+4z^2} dz$, where C is the circle $\|z-2-i\| = 2$.	**[8]**	**CO3**
8.	Solve the difference equation $y_n - 2y_{n-1} - 3y_{n-2} = 0, n \geq 2$ by the generating	**[8]**	**CO1**

	function method with initial conditions $y_0 = 3$ and $y_1 = 1$.		
9.	Show that $\int_{-1}^{1} x^2 P_{n-1}(x) P_{n+1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$	**[8]**	**CO2**
10.	Solve $\left(D^2 + DD' - 6D'^2\right)z = x^2 \sin(x+y)$. **OR** Solve $r - t = tan^3 x \tan y - \tan x\ tan^3 y$.	**[8]**	**CO5**
SECTION C **(Q11 is compulsory and Q12 have internal choice)**			
11.A	By the method of contour integration prove that $\int_0^{\infty} \frac{\cos x\, dx}{x^2+4} = \frac{\pi}{4e^2}$.	**[10]**	**CO4**
11.B	Prove that $\int_0^{2\pi} \frac{\cos 2\theta\, d\theta}{5+4\cos\theta} = \frac{\pi}{6}$	**[10]**	**CO4**
12.	A tightly stretched flexible string has its end fixed at $x = 0$ and $x = l$. At time $t = 0$, the string is given a shape defined by $F(x) = \mu x(l-x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at any time $t > 0$. **OR** The ends A and B of a rod of length L are maintained at temperatures 0°C and 100°C respectively until steady state conditions prevails. Suddenly the temperature at the end A is increased to 20°C and the end B is decreased to 60°C. Find the temperature distribution in the rod at time t.	**[20]**	**CO5**