Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme Name: B.Sc. (Honours) (Physics and Chemistry) **Semester : I**
Course Name : Generic-Electives I (Mathematics)-Matrices **Time : 03 Hrs.**
Course Code : MATH 1029 **Max. Marks : 100**
Nos. of page(s) : 02
Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (Each carrying 8 marks) and attempt all questions from **Section C** (each carrying 20 marks).

SECTION A
(Attempt all questions)

S. No.		Marks	CO
Q 1	Let A be a square matrix of order $n \times n$. Prove that if A is an idempotent matrix, then the $\det(A)$ is equal to either 0 or 1.	4	CO1
Q 2	Under what condition, the rank of the matrix $\begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 3?	4	CO2
Q 3	Show that the vectors $X_1 = (a_1,\ b_1)$ and $X_2 = (a_2,\ b_2)$ are linearly dependent if and only if $a_1 b_2 - a_2 b_1 = 0$.	4	CO3
Q 4	Form the matrix whose eigenvalues are $\alpha - 5$, $\beta - 5$ and $\gamma - 5$ where α, β, γ are the eigenvalues of the matrix $\begin{bmatrix} -1 & -2 & -3 \\ 4 & 5 & -6 \\ 7 & -8 & 9 \end{bmatrix}$.	4	CO4
Q 5	Suppose C is a 5×5 matrix all of whose eigenvalues are positive integers. If the determinant of C is 12 and the trace of C is 9 then find the characteristic polynomial of C.	4	CO4

SECTION B
(Q6,Q8, Q10 are compulsory and Q7 & Q9 has internal choice)

S. No.		Marks	CO
Q 6	Find the values of l, m and n if the matrix $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal. Also find A^{-1}.	8	CO1
Q 7	Investigate for what values of λ and μ the equations $x + 2y + z = 8;\ 2x + 2y + 2z = 13;\ 3x + 4y + \lambda z = \mu$ have **(i)** no solution **(ii)** unique solution, and **(iii)** many solutions. **OR** Show that the system of equations $3x + 4y + 5z = \alpha;\ 4x + 5y + 6z = \beta;\ 5x + 6y + 7z = \gamma$ is consistent only if α, β and γ are in arithmetic progression.	8	CO2

Q 8	Examine the following vectors for linear dependence or independence. If dependent, find the relation among them. $$X_1 = (1, -1, 3, 2),\ X_2 = (-2, -5, 2, 2), X_3 = (4, 3, 4, 2)$$	**8**	**CO3**
Q 9	Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and verify Cayley-Hamilton theorem for matrix A. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A. (Here I is the identity matrix.) **OR** Show that the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is not diagonalizable.	**8**	**CO4**
Q 10	Find the Characteristic and Minimal polynomial of the matrix $$A = \begin{bmatrix} 2 & 7 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{bmatrix}$$	**8**	**CO5**
SECTION-C **(Q 11 is compulsory and Q 12 has internal choice)**			
Q 11	**(a)** Suppose k is positive and the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & k & 3 \end{bmatrix}$ is such that $\det(A) = 1$. Consider the unique decomposition $A = LU$, where L is a lower triangular matrix and $U = L^T$, where L^T denotes the transpose matrix of L. Let $X \in \mathbb{R}^3$ *and* $b = [1,\ 1,\ 3]^t$. Find the solution of the system $AX = b$ where $X = [x,\ y,\ z]^t$. **(b)** In a given electrical network, the equations for the currents i_1, i_2, i_3 are given by $$2i_1 + 3i_2 + i_3 = 9;\ i_1 + 2i_2 + 3i_3 = 6;\ 3i_1 + i_2 + 2i_3 = 8.$$ Apply Crout's method to find the value of i_1, i_2, i_3.	**10+10**	**CO3**
Q12	Determine a diagonal matrix orthogonally similar to the real symmetric matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Also find the modal matrix. **OR** For a symmetric matrix A, the eigenvectors are $[1, 1, 1]^T, [1, -2, 1]^T$ corresponding to $\lambda_1 = 6$ and $\lambda_2 = 12$. Find the eigenvector corresponding to $\lambda_3 = 6$ and find the matrix A.	**20**	**CO4**

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SECTION A
(Attempt all questions)

S. No.		**Marks**	**CO**
Q 1	Let n be an odd positive integer and let K be an $n \times n$ skew symmetric matrix. Prove that K is singular.	**4**	**CO1**
Q 2	Find the values of x for which the rank of the matrix $\begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 2?	**4**	**CO2**
Q 3	If the vectors $(0,1,a)$; $(1,a,1)$ and $(a,1,0)$ are linearly dependent then find the value of a.	**4**	**CO3**
Q 4	Under what condition does the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $a,b,c,d \in \mathbb{R}$ have no real eigenvalues?	**4**	**CO4**
Q 5	Suppose C is a 6×6 matrix with eigenvalues 0, 1 and 3 of multiplicities 3, 2 and 1 respectively. Find the determinant of the matrix $2I - C$. (Here I is the identity matrix.)	**4**	**CO4**

SECTION B
(Q6,Q8, Q10 are compulsory and Q7 & Q9 has internal choice)

S. No.		**Marks**	**CO**
Q 6	Prove that the matrix $A = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is unitary and hence find A^{-1}.	**8**	**CO1**
Q 7	Investigate for what values of λ and μ the equations $x + y + z = 6;\ x + 2y + 3z = 10;\ x + 2y + \lambda z = \mu$ have **(i)** no solution **(ii)** unique solution, and **(iii)** many solutions. **OR** Examine the consistency of the following system and if consistent solve for x, y, z $-\frac{1}{x} + \frac{3}{y} + \frac{4}{z} = 30;\ \frac{3}{x} + \frac{2}{y} - \frac{1}{z} = 9;\ \frac{2}{x} - \frac{1}{y} + \frac{2}{z} = 10.$	**8**	**CO2**

Q 8	Are the vectors $$X_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, X_2 = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}, \quad X_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$ linearly dependent? If yes, then find a non-trivial dependence relationship among these vectors.	**8**	**CO3**
Q 9	Verify Cayley-Hamilton theorem for matrix $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$. Hence find A^{-1} if it exists. **OR** Show that the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$ is not diagonalizable.	**8**	**CO4**
Q 10	Find the Characteristic and Minimal polynomial of the matrix $$A = \begin{bmatrix} 4 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}.$$	**8**	**CO5**
SECTION-C **(Q 11 is compulsory and Q 12 has internal choice)**			
Q 11	**(a)** Solve the equation $AX = B$ where $$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$ by Choleski decomposition method. **(b)** In a given electrical network, the equations for the currents i_1, i_2, i_3 are given by $$2i_1 + 3i_2 + i_3 = 9; \ i_1 + 2i_2 + 3i_3 = 6; \ 3i_1 + i_2 + 2i_3 = 8.$$ Apply Doolittle's method to find the value of i_1, i_2, i_3.	**10+10**	**CO3**
Q12	Determine a diagonal matrix orthogonally similar to the real symmetric matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. Also find the model matrix. **OR** Find a matrix P which transform the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ to diagonal form. Hence find A^4.	**20**	**CO4**