Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2018

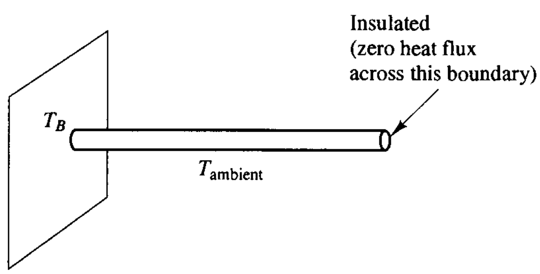
Course: Finite Volume Methods for conservation laws (ASEG7021) Semester: I
Programme: M. Tech CFD
Time: 03 hrs. Max. Marks: 100
Instructions: Make use of sketch/plots to elaborate your answer. All sections are compulsory.

SECTION A

S. No.		**Marks**	**CO**
Q 1.	Differentiate between Lagrangian and Eulerian descriptions. Give suitable example clearly stating advantages and disadvantages for each description.	**[04]**	**CO1**
Q 2.	Classify the steady two-dimensional velocity potential equation: $$\left(1-M^2\right)\phi_{xx}+\phi_{yy}=0$$ where *M* is mach number. Explain the physical meaning of various classifications based on *M*.	**[04]**	**CO2**
Q 3.	Explain the algorithm of the Jacobi Iteration method applied to a parabolic partial differential equation.	**[04]**	**CO2**
Q 4.	Define the following terms: a) Diffusion number b) Approximate factorization	**[04]**	**CO1**
Q 5.	What is the stability requirement of an explicit finite difference equation produced from the model equation $\frac{\partial u}{\partial t}=\propto\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x^2}\right]$.	**[04]**	**CO3**

SECTION B

S. No.		**Marks**	**CO**
Q 6.	Explain the basic methodology of Finite Difference Method. Derive the discretization equation with the help of Taylors series for the following methods, • Forward Difference • Backward Difference • Central Difference State the accuracy and stability criteria for each method.	**[10]**	**CO1**
Q 7.	Derive the explicit MacCormack time marching algorithm for the solution of	**[10]**	**CO2**

	transient Euler equations in 2-dimensions		
Q 8.	Define the UPWIND interpolation scheme for the evaluation of fluxes at face centre using the nodal values on a structured finite volume grid. Find an expression for the artificial diffusivity introduced by this scheme.	**[10]**	**CO3**
Q 9.	Describe the underlying concept of the following terminologies: a) Consistency b) Stability c) Convergence d) Conservative property	**[10]**	**CO3**
	SECTION-C		
Q 10.	Shown in Figure below is a cylindrical fin with uniform cross-sectional area A. The base is at a temperature of 100 °C (T_B) and the end is insulated. The fin is exposed to an ambient temperature of 20 °C. One-dimensional heat transfer in this situation is governed by $$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - hP(T - T_\infty) = 0$$ where h is the convective heat transfer coefficient, P the perimeter, k the thermal conductivity of the material and T_∞ the ambient temperature.  Calculate the temperature distribution along the fin and compare the results with the analytical solution given by $$\frac{T - T_\infty}{T_B - T_\infty} = \frac{\cosh[n(L-x)]}{\cosh(nL)}$$ where $n^2 = hP/(kA)$, L is the length of the fin and x the distance along the fin. Data: L = 1 m, hP/{kA) = 25 m^{-2} (note kA is constant). Solve for 5 and 10 nodes. Compare the result.	**[20]**	**CO4**
Q 11.	Consider the model equation:		

$$a\frac{\partial u}{\partial x}=\vartheta\frac{\partial^2 u}{\partial y^2}$$

(a) Write an explicit formulation using a first-order forward differencing in *x* and a second-order central differencing in *y*.

(b) Use von Neumann stability analysis to determine the stability requirement of the scheme.

OR

A property ∅ is transported by means of convection and diffusion through the one-dimensional domain sketched in figure below. The governing equation below;

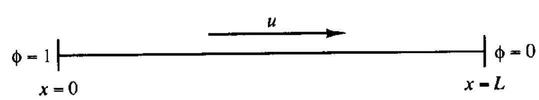
$$\frac{d}{dx}(\rho u \varnothing)=\frac{d}{dx}\left(\tau\frac{d\varnothing}{dx}\right)$$

boundary conditions are $\varnothing_0 = 1$ at x = 0 and $\varnothing_L = 0$ at x = L . Using five equally spaced cells (for first two cases) and the central differencing scheme for convection and diffusion calculate the distribution of ∅ as a function of x for cases:

(i) Case 1: u = 0.1 m/s, using 5 cells

(ii) Case 2: u = 2.5 m/s, using 5 cells

(iii) Case 3: u = 2.5 using 10 cells



and compare the results with the analytical solution given below. The following data apply: length L = 1.0 m, p = 1.0 kg/m3, Γ = 0.1 kg/m/s.

$$\frac{\phi-\phi_0}{\phi_L-\phi_0}=\frac{\exp(\rho u x/\Gamma)-1}{\exp(\rho u L/\Gamma)-1}$$

[20] **CO4**

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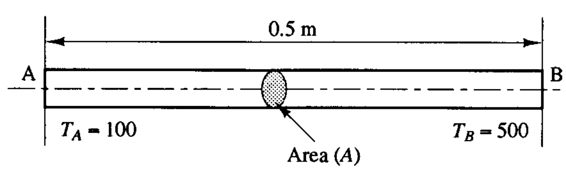
SECTION A

S. No.		Marks	CO
Q 1.	Under what conditions, the Navier-Stokes equation will become, • Elliptic • Parabolic • Hyperbolic	[04]	CO2
Q 2.	Differentiate between explicit and implicit methods for converting partial differential equations into finite differential equations.	[04]	CO1
Q 3.	Find a forward difference approximation of $O(\Delta x)$ for $\frac{\partial^4 f}{\partial x^4}$	[04]	CO2
Q 4.	What do you mean by initial and boundary conditions? Define various types of boundary conditions which are usually encountered in CFD problems.	[04]	CO1
Q 5.	What is the importance of CFL condition? Relate it to the analysis of stability?	[04]	CO3

SECTION B

Q 6.	Explain with proper example how a pentadiagonal coefficient matrix can be reduced to two sets of tridiagonal coefficient matrix to be solved in sequence.	[10]	CO1
Q 7.	Determine an approximate backward difference representation for $\frac{\partial^3 f}{\partial x^3}$ which is of order (Δx), given evenly spaced grid points by means of: (a) Taylor series expansions. (b) A backward difference reccurence formulae (c) A third-degree polynomial passing throug four points.	[10]	CO2
Q 8.	Given the function $f(x)=\frac{1}{4}x^2$, compute the first derivative of f at $x=2$ using forward and backward differencing of order (Δx). Compare the results with a central differencing of $O(\Delta x)^2$ and the exact analytical value. Repeat the computations for a step size of 0.4.	[10]	CO3

Q 9.	Explain the various methods for the approximation of surface integrals over a 2 - dimensional control volume. **OR** Derive the explicit Mac-Cormack time marching algorithm for the solution of transient Euler equations in 2-Dimensions.	**[10]**	**CO3**
SECTION C			
Q 10.	Given the following data, compute $f'(5)$, $f'(7)$ and $f'(9)$. Use finite difference of order (Δx). Compare the results to the values obtained by finite differencing of order $(\Delta x)^2$. x: 5, 6, 7, 8, 9 $f(x)$: 25, 36, 49, 64, 81	**[20]**	**CO4**
Q 11.	Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of 100 °C and 500 °C respectively. The one-dimensional problem sketched in Figure below, is governed by $$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$ Calculate the steady state temperature distribution in the rod. Thermal conductivity k equals 1000 W/m/K, cross-sectional area A is 10 x 10^{-3} m^2. Use at least 5 control volumes with appropriate interpolation scheme.  	**[20]**	**CO4**

x	5	6	7	8	9
$f(x)$	25	36	49	64	81