Roll No: ---------------------------

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme: B.Tech. (APE-GAS, ME,MECHATRONICS,ADE, Chemical) **Semester – I**
Course Name: Mathemactics-III **Max. Marks : 100**
Course Code: MATH-2003 **Duration : 3 Hrs**
No. of page/s: 02

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks); attempt all questions from **Section C** (each carrying 20 marks).

SECTION A
(Attempt all questions)

1.	Verify that $X = \begin{pmatrix} 2e^{5t} \\ e^{5t} \end{pmatrix}$ is the solution of the system $\frac{dX}{dt} = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix} X$.	**[4]**	**CO1**
2.	Use Cauchy integral formula to evaluate $\oint_C \frac{e^{1/z}}{z} dz$, where the contour C is the circle $\lvert z \rvert = 1$ traced counterclockwise.	**[4]**	**CO2**
3.	Use Cauchy Residue theorem to evaluate $\oint_C \frac{P(z)}{(z-1)^{15}} dz$, where $P(z) = \sum_{n=1}^{15} z^n$ and the contour C is $z = e^{i2\theta}, 0 \le \theta \le 2\pi$.	**[4]**	**CO3**
4.	Let C be a closed differentiable contour oriented counterclockwise and let $\oint_C \bar{z}\, dz = A$ where $z = x + iy$. Evaluate $\oint_C (x+y)\, dz$ in terms of A.	**[4]**	**CO3**
5.	Find the particular integral of $(D^2 - 2DD' + D'^2)z = e^{x+2y}$.	**[4]**	**CO4**

SECTION B
(Q6-Q8 are compulsory and Q9 has internal choice)

6.	Find the general and singular solution of Clairaut's equation $y = px - e^p$.	**[10]**	**CO1**
7.	Supose $\phi_k(t) = \begin{bmatrix} \phi_{1k}(t) \\ \phi_{2k}(t) \\ . \\ . \\ . \\ \phi_{nk}(t) \end{bmatrix}$, $k = 1,2,3, \dots, n$ are the solutions of $\frac{dX}{dt} = AX$. If the Wronskian $W(t_0) = 0$ at some $t_0 \in [a, b]$ then prove that ϕ_k , $k = 1,2,3, \dots, n$ are linearly dependent on $a \le t \le b$.	**[10]**	**CO1**

8.	If $u(x,y)=\frac{1}{2}\log_e(x^2+y^2)$, find $v(x,y)$ such that $f(z)=u(x,y)+iv(x,y)$ is analytic by using Milne Thomson method.	**[10]**	**CO2**
9.	If a is any complex number with $\lvert a\rvert<1$ and C is the simple closed curve $\lvert z\rvert=1$ oriented counterclockwise, then find the value of $$\frac{(1-\lvert a\rvert^2)}{\pi}\int_C \frac{\lvert dz\rvert}{\lvert z+a\rvert^2}$$ **OR** If $P(z)$ is a polynomial and C denotes the circle $\lvert z-a\rvert=R$. What is the value of $\oint_C P(z)d\bar{z}$?	**[10]**	**CO3**
SECTION C (Q10 is compulsory and Q11 has internal choice)			
10.	**a**. Construct a linear fractional transformation that maps the points $0,-1$ and ∞ onto the points $-1,-2-i$ and i respectively. **b**. Find the solution of the form $u=f_1(x)\,f_2(y)$ satisfying $\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}-u=0$ with $u=6e^{-3x}$ when $y=0$.	**[10+10]**	**CO3** **CO4**
11.	**a**. Consider the line L defined by the equation $z=(1-i)x$ (where x is real) which divides the $z-$plane into two symmetrical halves S_1 and S_2. If S_1 contains the positive quadrant then find the image of S_1 under the bilinear transformation $w(z)=\frac{z-1}{z+i}$. **b**. A string is stretched between two fixed points at a distance l apart. Motion is started by displacing the string in the form $y=y_0\sin\frac{\pi x}{l}$ from which it is released at time $t=0$. Find the displacement at any point at a distance x from one end at time t. **OR**	**[10+10]**	**CO3** **CO4**
11.	**a**. Consider the functions $f(z)=\frac{z^2+\alpha z}{(z+1)^2}$ and $g(z)=\sinh\left(z-\frac{\pi}{2\alpha}\right),\alpha\neq 0$. If the residue of $f(z)$ at its pole is equal to 1, then find a point where the function $g(z)$ is not conformal. **b**. If the mid point of a string of length l with fixed end points $x=0$ and $x=l$ is taken to a small height h and released from the rest at time $t=\mathbf{0}$. Then find the displacement function $y(x,t)$.	**[10+10]**	**CO3** **CO4**

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Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

SECTION A
(Attempt all questions)

1.	Show that $X = \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix}$ will satisfy the system $\frac{dX}{dt} = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix} X$.	**[4]**	**CO1**
2.	Evaluate $\oint_C \frac{\cos\left(\frac{1}{z}\right)}{z} dz$, where the contour C is the circle $\lvert z \rvert = 1$ traced counterclockwise, using Cauchy's integral formula	**[4]**	**CO2**
3.	Evaluate $\oint_C (x-y)\,dz$ in terms of A, where C be a closed differentiable contour oriented counterclockwise and let $\oint_C \bar{z}\ dz = A$ where $z = x + iy$.	**[4]**	**CO3**
4.	Find the particular integral of $(D^2 - 2DD' + D'^2)z = e^{x-2y}$.	**[4]**	**CO4**
5.	Evaluate $\oint_C \frac{P(z)}{(z-1)^{15}} dz$, where $P(z) = \sum_{n=1}^{15} z^n$ and the contour C is $z = e^{i\theta}, 0 \le \theta \le 4\pi$ using Cauchy's Residue theorem.	**[4]**	**CO3**

SECTION B
(Q6-Q8 are compulsory and Q9 has internal choice)

6.	Find the general and singular solution of the equation $y = 4px^2 - 2e^{2p}$.	**[10]**	**CO1**
7.	If $v(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$, find $v(x,y)$ such that $f(z) = u(x,y) + iv(x,y)$ is analytic by using Milne Thomson method.	**[10]**	**CO2**

8.	Consider the system $\frac{dX}{dt} = AX$. If $\phi_k(t) = \begin{bmatrix} \phi_{1k}(t) \\ \phi_{2k}(t) \\ . \\ . \\ . \\ \phi_{nk}(t) \end{bmatrix}$, $k = 1,2,3, \dots, n$ are the solutions of this system. Then prove that if the Wronskian $W(t_0)$ vanishes at some $t_0 \in [a, b]$ then ϕ_k, $k = 1,2,3, \dots, n$ are linearly dependent on $t \in [a, b]$.	**[10]**	**CO1**
9.	If C is the simple closed curve $\|z\| = 1$ oriented counterclockwise, b is any complex number with $\|b\| < 1$ then find the value of $$\frac{\left(1-\|b\|^2\right)}{\pi}\int_C \frac{\|dz\|}{\|z-b\|^2}$$ **OR** Evaluate $\oint_C P(z)d\bar{z}$ where, $P(z)$ is a polynomial and C denotes the circle $\|z+b\| = R$.	**[10]**	**CO3**
SECTION C **(Q10 is compulsory and Q11 has internal choice)**			
10.	**a**. Find the functions $f_1(x)$ and $f_2(x)$ such that $u = f_1(x)\, f_2(y)$ is the solution of $\frac{\partial u}{\partial x} - 4\frac{\partial u}{\partial y} - u = 0$ with $u(x, 0) = 6e^{-3x}$. **b**.Construct a bilinear transformation that maps the points 0,1 and ∞ onto the points $-1, 2+i$ and i respectively.	**[10+10]**	**CO3** **CO4**
11.	**a**. Consider the line L defined by the equation $z = (1-i)x$ (where x is real) which divides the $z-$plane into two symmetrical halves S_1 and S_2. If S_1 contains the negative quadrant then find the image of S_1 under the bilinear transformation $w(z) = \frac{z-1}{z+i}$. **b**. A string is stretched between two fixed points at a distance $2l$ apart. Motion is started by displacing the string in the form $y = y_0 \sin\frac{\pi x}{2l}$ from which it is released at time $t = 0$. Find the displacement at any point at a distance x from one end at time t. **OR**	**[10+10]**	**CO3** **CO4**

11.	**a**. Consider the functions $f(z) = \frac{z^3+\alpha z}{(z+1)^2}$ and $g(z) = \sinh\left(z - \frac{\pi}{2\alpha}\right), \alpha \neq 0$. If the residue of $f(z)$ at its pole is equal to 6, then find a point where the function $g(z)$ is not conformal. **b**. If the mid point of a string of length $2l$ with fixed end points $x = 0$ and $x = 2l$ is taken to a small height h and released from the rest at time $t = \mathbf{0}$. Then find the displacement function $y(x,t)$.	**[10+10]**	**CO3** **CO4**