

Roll No: -----



**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**

End Semester Examination, December 2018

Programme: B. Tech. [ME+ME(MD)+ME(MSNT)+ME(PROE)+ME(THE)]

Semester – V

Course Name: Applied Numerical Techniques

Max. Marks:100

Course Code: MATH 305

Duration: 3 Hrs

No. of page/s: 02

**Instructions:**

Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks); attempt **Section C** (each carrying 20 marks).

**Section A**

1.	Show that the bisection method always converges for the equation $f(x) = 0$ .	[4]	CO2										
2.	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ , by using Simpson's 1/3 rule.	[4]	CO1										
3.	Discuss the method of point collocation with an example.	[4]	CO4										
4.	Classify the following equation $\frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$	[4]	CO3										
5.	Find $f(3)$ and $f'(3)$ from <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>x</math></td> <td>0</td> <td>1</td> <td>2</td> <td>5</td> </tr> <tr> <td><math>f(x)</math></td> <td>2</td> <td>3</td> <td>12</td> <td>147</td> </tr> </table>	$x$	0	1	2	5	$f(x)$	2	3	12	147	[4]	CO1
$x$	0	1	2	5									
$f(x)$	2	3	12	147									

**SECTION B**

6.	Find an approximate solution of the differential equation by method of least squares $\frac{d^2 y}{dx^2} - u = x, x \in [0, 1]$ with boundary conditions $u(0) = u(1) = 0$ . Use only two basis functions.	[10]	CO4
7.	Solve by Gauss-Seidel method (two approximations) $6x + y + z = 105$ $4x + 8y + 3z = 155$ $5x + 4y - 10z = 65$	[10]	CO2
8.	Find the value of $y$ for $x=0.1$ by Picard's method, given that $y(0)=1$ and	[10]	CO3

	$\frac{dy}{dx} = \frac{y-x}{y+x}$																								
9.	<p>Derive the newton Cotes formula for integration.</p> <p style="text-align: center;"><b>OR</b></p> <p>The velocity (<math>v</math>) of a car which starts from rest is given at fixed intervals of time (<math>t</math>) as follows.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>t</math></td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> <td>12</td> <td>14</td> <td>16</td> <td>18</td> <td>20</td> </tr> <tr> <td><math>v</math></td> <td>10</td> <td>18</td> <td>25</td> <td>29</td> <td>32</td> <td>20</td> <td>11</td> <td>5</td> <td>2</td> <td>0</td> </tr> </table> <p>Estimate approximately the distance covered by car in 20 units of time.</p>	$t$	2	4	6	8	10	12	14	16	18	20	$v$	10	18	25	29	32	20	11	5	2	0	[10]	CO1
$t$	2	4	6	8	10	12	14	16	18	20															
$v$	10	18	25	29	32	20	11	5	2	0															
<b>SECTION C</b>																									
9.A	<p>Using Gauss Elimination Method Solve the following system of equations</p> $2x + 4y + z = 3$ $3x + 2y - 2z = -2$ $x - y + z = 6$	[10]	CO2																						
9.B	<p>Apply Runge-Kutta method (fourth order) to find an approximate value of <math>y(0.1)</math>, given that</p> $\frac{dy}{dx} = x + y$ <p style="text-align: center;"><math>y(0)=1</math> with <math>h=0.1</math></p>	[10]	CO3																						
10. A	<p>Derive the Lagrange's interpolation formula for interpolation.</p> <p style="text-align: center;"><b>OR</b></p> <p>Derive Newton's forward difference formula for interpolation.</p>	[10]	CO1																						
10.B	<p>Solve the boundary value problem defined by</p> $y'' - x = 0 \quad \text{and} \quad y(0)=1, \quad y'(1)=-1/2$ <p style="text-align: center;"><b>OR</b></p> <p>Solve the equation <math>y'' + y = -x</math>, <math>0 &lt; x &lt; 1</math> and <math>y(0)=y(1)=0</math></p>	[10]	CO4																						

Roll No: -----



**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**

End Semester Examination, December 2018

Programme: B. Tech. [ME+ME(MD)+ME(MSNT)+ME(PROE)+ME(THE)]

Semester – V

Course Name: Applied Numerical Techniques  
100

Max. Marks:

Course Code: MATH 305  
Hrs

Duration: 3

No. of page/s: 02

**Instructions:**

Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks); attempt **Section C** (each carrying 20 marks).

**Section A**

1.	Show that the fixed point iteration method may not converge for some equations $f(x)=0$ .with the help of an example	[5]	CO2										
2.	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's 3/8 rule.	[5]	CO1										
3.	Discuss the method of point collocation with an example.	[5]	CO4										
4.	Derive the Bessel's formula for the interpolation.	[5]	CO3										
5.	Find $f(0.5)$ where <table border="1" style="display: inline-table; margin-left: 20px;"> <tr> <td><math>x</math></td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td><math>f(x)</math></td> <td>1</td> <td>2</td> <td>1</td> <td>10</td> </tr> </table>	$x$	0	1	2	3	$f(x)$	1	2	1	10	[5]	CO1
$x$	0	1	2	3									
$f(x)$	1	2	1	10									

**SECTION B**

6.	Discuss the method of least squares with an example.	[10]	CO4
7.	Solve by Gauss-Jacobi method (two approximations) $6x + y + z = 105$ $4x + 8y + 3z = 155$ $5x + 4y - 10z = 65$	[10]	CO2
8.	Find the value of $y$ for $x=0.1$ by Taylor Series method, given that $y(0)=1$ and $\frac{dy}{dx} = \frac{y-x}{y+x}$	[10]	CO3

9.	<p>Derive the Trapezoidal rule by deriving Newton-Cotes formula for integration.</p> <p style="text-align: center;"><b>OR</b></p> <p>The velocity (<math>v</math>) of a car which starts from rest is given at fixed intervals of time (<math>t</math>) as follows.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>t</math></td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> <td>12</td> <td>14</td> <td>16</td> <td>18</td> <td>20</td> </tr> <tr> <td><math>v</math></td> <td>10</td> <td>18</td> <td>25</td> <td>29</td> <td>32</td> <td>20</td> <td>11</td> <td>5</td> <td>2</td> <td>0</td> </tr> </table> <p>Estimate approximately the distance covered by car in 20 units of time.</p>	$t$	2	4	6	8	10	12	14	16	18	20	$v$	10	18	25	29	32	20	11	5	2	0	[10]	CO1
$t$	2	4	6	8	10	12	14	16	18	20															
$v$	10	18	25	29	32	20	11	5	2	0															

**SECTION C**

9.A	<p>Using LU (Crout's) decomposition Method Solve the following system of equations</p> $2x + 4y + z = 3$ $3x + 2y - 2z = -2$ $x - y + z = 6$	[10]	CO2
-----	--	------	-----

9.B	<p>Apply Euler's method to find an approximate value of <math>y(0.1)</math>, given that</p> $\frac{dy}{dx} = x + y$ $y(0) = 1$	[10]	CO3
-----	--	------	-----

10.A	<p>Derive the Newton's interpolation formula (divided difference) and hence find <math>f(3)</math></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>x</math></td> <td>0</td> <td>1</td> <td>2</td> <td>5</td> </tr> <tr> <td><math>f(x)</math></td> <td>2</td> <td>3</td> <td>12</td> <td>147</td> </tr> </table> <p style="text-align: center;"><b>OR</b></p> <p>Derive Newton's backward difference formula for interpolation.</p>	$x$	0	1	2	5	$f(x)$	2	3	12	147	[10]	CO1
$x$	0	1	2	5									
$f(x)$	2	3	12	147									

10.B	<p>Solve the BVP <math>y'' - 64y + 10 = 0</math> with <math>y(0) = y(1) = 0</math>.</p> <p style="text-align: center;"><b>OR</b></p> <p>Solve the equation <math>y'' + y = -x</math>, <math>0 &lt; x &lt; 1</math> and <math>y(0) = y(1) = 0</math></p>	[10]	CO4
------	---	------	-----