Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme Name: M. Tech. CFD **Semester : I**
Course Name : Introduction to CFD **Time : 03 hrs.**
Course Code : ASEG 7001 **Max. Marks: 100**
Nos. of page(s) : 03
Instructions: Assume any missing data appropriately.

SECTION A

S. No.		Marks	CO
Q 1	List the various physical boundary conditions encountered in a non-isothermal fluid flow.	4	CO1
Q 2	When the law of conservation of mass is applied to finite control volume Ω, moving with flow, the governing equation is given by $\frac{D}{Dt}\oiiint \rho \, d\Omega = 0$ Convert this equation to divergence form, applicable to finite control volume fixed in space.	4	CO1
Q 3	Sketch the various models of fluid flow used for derivation of governing equations. Write down the forms of equations that emanate from these models on applications conservation laws.	4	CO1
Q 4	Illuminate the need of a body fitted coordinate system for the solution of governing flow equations using finite difference method.	4	CO3
Q 5	Compare with illustrations, the explicit and implicit schemes for solution of partial differential equations.	4	CO2
	SECTION B		
Q 6	Apply the law of conservation of momentum for an infinitesimally small element of a viscous fluid moving in space and hence deduce the momentum equation for fluids in non- conservation form.	10	CO1
Q 7	Consider the following system of equations	10	CO2

	$$\frac{\partial u_1}{\partial t} + \frac{\partial u_2}{\partial x} = 0,$$ $$\frac{\partial u_2}{\partial t} + \frac{\partial u_3}{\partial x} = 0,$$ $$\frac{\partial u_3}{\partial t} + 4\frac{\partial u_1}{\partial x} - 17\frac{\partial u_2}{\partial x} + 8\frac{\partial u_3}{\partial x} = 0.$$ Classify this system of equations as hyperbolic or elliptic, based on the eigenvalue method.		
Q 8	Analyze the stability of explicit FTCS scheme for solution of transient, one-dimensional heat conduction equation and hence establish the stability criterion. Assume the error to be of the form $$\epsilon_m(x,t) = e^{at}e^{ik_m x}$$ ***OR*** An explicit scheme for solving the first-order wave equation is given by: $$u_j^{n+1} = u_j^n - c\frac{\Delta t}{\Delta x}\left(\frac{u_{j+1}^{n+1} - u_{j-1}^{n+1}}{2}\right)$$ Apply the Fourier stability analysis to this scheme, and determine the stability restrictions, if any.	**10**	**CO4**
Q 9	Discuss an explicit time marching algorithm for the solution of transient Euler equations in 2-dimensions.	**10**	**CO2**
	SECTION-C		
Q 10	Consider a two-dimensional square plate ABCD with edges AB and CD maintained at temperatures of 200 ^oK and 100 ^oK respectively. The other two edges DA and BC are also maintained at temperatures of 200 ^oK, except at the corners C and D. Find the steady state temperatures of at least 9 locations on the plate. Take *AB=BC=CD=DA=* 4 cm. Use pure Gauss-Seidel relaxation scheme for at least 4 iterations. The two-dimensional steady state heat conduction is governed by $$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$ ***OR***	**20**	**CO5**

	Consider a 6 cm long steel, whose left and right ends are kept at fixed temperatures of 0 ^{0}C and 100 ^{0}C respectively. The initial (t = 0 s) temperature at other locations on the rod is 25 ^{0}C. Predict the temperature at any five locations on the rod after 5 seconds (t =5 s) using an explicit Forward in Time and Central in Space (FTCS) finite difference scheme. Hint: Take Δx=0.01 m.		
Q 11	Derive the *modified equation* that results from the first order forward in time and backward in space discretization of the first order wave equation. $$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$ Discuss the nature of dominating error for the above discretization and suggest means to minimize them.	**20**	**CO4**

Name: **Enrolment No:**	UPES

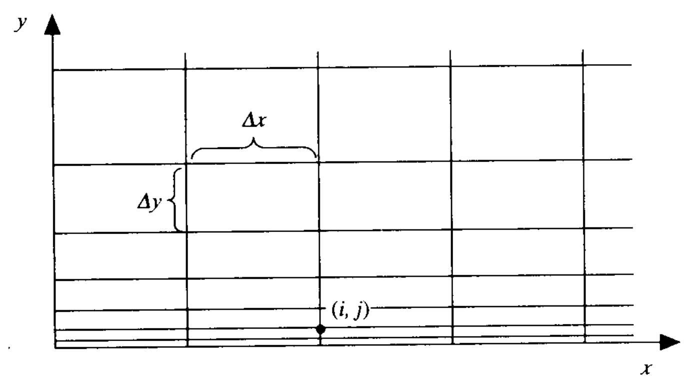
UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme Name: M. Tech. CFD **Semester : I**
Course Name : Introduction to CFD **Time : 03 hrs.**
Course Code : ASEG 7001 **Max. Marks: 100**
Nos. of page(s) : 03
Instructions: Assume any missing data appropriately.

SECTION A

S. No.		Marks	CO
Q 1	Write down second order accurate finite difference stencils for discretization of the following derivatives. a. $\frac{\partial^2 u}{\partial y^2}$ b. $\frac{\partial u}{\partial y}$	**4**	**CO1**
Q 2	Consider the viscous flow of air over a flat plate. At a given station in the flow direction, the variation of the flow velocity, *u*, in the direction perpendicular to the plate (the y direction) is given at discrete grid points equally spaced in y direction with Δy = 2.54 mm. **y (mm)** **u (m/s)** 0 0 2.54 45.72 5.08 87.41 7.62 125.0 Imagine that the values of ***u*** listed above are discrete values at discrete grid points located at ***y*** = 0, 2.54, 5.08 and 7.62 mm the same nature as would be obtained from a numerical finite difference solution of the flow field. For viscosity coefficient, μ =1.7895 x 10^{-5} kg/m-s, using these discrete values; Calculate the shear stress at the wall $\boldsymbol{\tau_w}$ three different ways, namely: a. Using a first order one sided difference b. Using the second order one sided difference	**4**	**CO3**
Q 3	Define *numerical diffusion and dispersion.* Discuss the effect of numerical diffusion	**4**	**CO4**

	and dispersion on the solution of the one-dimensional scalar wave equation using the explicit Forward in Time and Backward in Space (FTBS) scheme. Suggest methods to alleviate the diffusive error.		
Q 4	Find the values of Mach number (M_∞) for which the system of equations given below is hyperbolic. $(1-M_\infty^2)\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0$ $\frac{\partial u'}{\partial y}-\frac{\partial v'}{\partial x}=0$	**4**	**CO1**
Q 5	Elucidate the need of grid and equation transformation for the solution flow over complex geometries using finite difference method.	**4**	**CO3**
SECTION B			
Q 6	Consider the numerical solution of steady viscous flow over a flat plate using a finite difference scheme. To calculate the details of this flow near the surface, very fine mesh, stretched in transverse direction is required as shown in figure below.  The solution requires an equispaced Cartesian grid in computational plane (ξ, η) which can be obtained through following direct transformations $\xi=x$ $\eta=\ln(1+y)$ If the continuity equation for above flow in physical plane (*x, y*) is $\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0$, find the continuity equation that is required to be solved in the computational plane.	**10**	**CO3**
Q 7	Consider the 2-dimensional transient heat conduction equation given below. The	**10**	**CO2**

	Crank-Nicolson discretization of the equation results in a pentadiagonal system of equations. Demonstrate an algorithm to solve the system of equations iteratively. $$\frac{\partial T}{\partial t}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$		
Q 8	Discuss the solution of Laplace equation in 2-dimensions using an explicit five point Gauss-Seidel Scheme point iterative scheme. Suggest modifications that can accelerate this scheme.	**10**	**CO2**
Q 9	Derive a third order, one-sided finite difference discretization of first order derivative $\frac{\partial u}{\partial x}$. ***OR*** Derive a fourth order accurate finite difference stencil for the mixed derivative $\frac{\partial^2 u}{\partial x^2}$	**10**	**CO2**
SECTION-C			
Q 10	Deduce the *modified equation* for the solution of the first order wave equation using Lax Method given by $$\frac{u_j^{n+1}-(u_{j+1}^n+u_{j-1}^n)/2}{\Delta t}+c\frac{u_{j+1}^n-u_{j-1}^n}{2\,\Delta x}=0$$ Hence, discuss the effect of the dominating error on the solution obtained.	**20**	**CO4**
Q 11	Consider the Couette flow between two plates, characterized by the parabolic equation, $$\frac{\partial u}{\partial t}-\nu\frac{\partial^2 u}{\partial y^2}=0, \quad \nu=0.000217\,\mathrm{m^2/s}$$ with initial and boundary conditions given as: Initial conditions at $t=0$ $\begin{cases} u=u_0=40\,\mathrm{m/s}, & y=0 \\ u=0, & 0<y\le h \end{cases}$ Boundary conditions at $t>0$ $\begin{cases} u=u_o=40\,\mathrm{m/s}, & y=0 \\ u=0, & y=h \end{cases}$ Evaluate the velocity at the midway location between the plates, at $t = 0.4$ seconds using Crank-Nicolson Scheme. Take $h = 0.04$ m and use $\Delta y = 0.004$ m.	**20**	**CO5**

	OR Consider square plate PQRS whose edges PQ, QR and SP are maintained at temperature of 400 ºK whereas the edge RS is maintained at 200 ºK. Find the steady state temperatures of at least 9 locations on the plate. Take *PQ*=*QR*=*RS*=*SP*= 4 cm. Use a point iterative relaxation scheme for at least 4 iterations with an over-relaxation factor of 1.2. The two-dimensional steady state heat conduction is governed by $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$		