UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme: B. Tech. (CE+RP, APE UP, APE GAS, GSE, GIE, Mining, FSE) **Semester: I**

Course Name: Mathematics I **Max. Marks : 100**

Course Code: MATH 1010 **Duration : 3 Hrs.**

No. of page/s: 02

Instructions:

Attempt all questions from **Section A** (each carrying 4 marks); all questions from **Section B** (each carrying 8 marks) and all questions from **Section C** (carrying 20 marks)**.**

Section A **(Attempt all questions)**			
1.	If $y = \sin nx + \cos nx$, prove that $\frac{d^r y}{dx^r} = n^r[1 + (-1)^r \sin 2nx]^{\frac{1}{2}}$.	**[4]**	**CO2**
2.	If 4, -7 and 3 are the Eigen values of a matrix $[A]_{3\times 3}$, then find the trace and the determinant of the matrix.	**[4]**	**CO1**
3.	Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$.	**[4]**	**CO3**
4.	Find the divergence and curl of the vector $\vec{V} = xyz\,\hat{\imath} + 3x^2y\,\hat{\jmath} + (xz^2 - y^2z)\hat{k}$.	**[4]**	**CO3**
5.	Find the coefficient a_0 for $f(x) = sin^5x$ from $x = -\pi$ to $x = \pi$.	**[4]**	**CO4**
SECTION B **(Q6-Q8 are compulsory. Q9 and Q10 have internal choices)**			
6.	Using Cayley-Hamilton Theorem find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.	**[8]**	**CO1**
7.	Taking vertical strip, evaluate $\iint_R f(x, y)\, dx\, dy$ over the rectangle $R = [0, 1; 0, 1]$ where $f(x, y) = \begin{cases} x + y, & \text{if } x^2 < y < 2x^2 \\ 0, & \text{otherwise} \end{cases}$.	**[8]**	**CO2**
8.	Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dydx$ by changing into polar coordinates.	**[8]**	**CO2**

9.	Evaluate $\iint_R \; x^2 dxdy$, where R is the region in the first quadrant bounded by $xy = 16, \;\; x = y, \; y = 0$ and $x = 8$. **OR** Evaluate $\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_0^{\sqrt{(1-x^2-y^2)}} \frac{1}{\sqrt{(1-x^2-y^2-z^2)}} dzdydx.$	**[8]**	**CO2**
10.	Show that the force field $\vec{F}$ given by $\vec{F} = 2xyz^2\,\hat{\imath} + (x^2z^2 + z\cos yz)\hat{\jmath} + (2x^2yz + y\cos yz)\hat{k}$ is irrotational. Find the scalar potential and the work done by $\vec{F}$ from any path from $(0,0,1)$ to $\left(1, \frac{\pi}{4}, 2\right)$. **OR** Using Green's theorem, evaluate $\int_C \; (x^2ydx + x^2dy)$ where C is the boundary described counter clockwise of the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$.	**[8]**	**CO3**
SECTION C **(Q11 is compulsory. Q12A and Q12B have internal choices)**			
11.A	Evaluate $\iint_S \; \vec{A}.\hat{n}\, dS$, where $\vec{A} = z\,\hat{\imath} + x\,\hat{\jmath} - 3y^2z\,\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.	**[10]**	**CO3**
11.B	Obtain the Fourier series of to represent $f(x) = x^2, -\pi < x < \pi$. Sketch the graph of $f(x)$.	**[10]**	**CO4**
12.A	Apply Green's theorem to evaluate $\int_C \; [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the area enclosed by the x axis and the upper half of the circle $x^2 + y^2 = a^2$ **OR** Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative force field . Find the scalar potential. Find also the work done in moving an object in this field from $(1,2,-1)$ to $(3,1,4)$.	**[10]**	**CO3**
12.B	Find the Fourier series to represent the function $f(x)$ given by $f(x) = \begin{cases} x & for \quad 0 \le x \le \pi \\ 2\pi - x & for\; \pi \le x \le 2\pi \end{cases}.$ **OR** Test the convergence of the following series: (i) $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \; \ldots\ldots\ldots\infty$ (ii) $\frac{1}{4.7.10} + \frac{4}{7.10.13} + \frac{9}{10.13.16} + \; \ldots\ldots\ldots\infty$	**[10]**	**CO4**

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	Section A **(Attempt all questions)**		
1.	If $y = \sqrt{(x+a)}$, find $\frac{d^n y}{dx^n}$.	**[4]**	**CO2**
2.	Obtain the Eigen values of A^3 where $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$.	**[4]**	**CO1**
3.	Find a unit vector normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.	**[4]**	**CO3**
4.	If $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$, find $\nabla.\vec{r}$ and $\nabla \times \vec{r}$.	**[4]**	**CO3**
5.	Find the coefficient a_0 for $f(x) = sin^3x\, cos^2x$ from $x = -\pi$ to $x = \pi$.	**[4]**	**CO4**
	SECTION B **(Q6-Q8 are compulsory. Q9 and Q10 have internal choices)**		
6.	Employing elementary row transformations, find the inverse of the matrix $$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$	**[8]**	**CO1**
7.	Evaluate $\iint_R e^{x+y} dx\, dy$, where R is the region which lies between two squares of sides 2 and 4 with center at the origin and sides parallel to the axes.	**[8]**	**CO2**
8.	Evaluate $\int_0^2 \int_0^{\sqrt{(2x-x^2)}} \frac{xdydx}{\sqrt{(x^2+y^2)}}$ by changing to polar coordinates.	**[8]**	**CO2**

9.	Using the transformation $x-y=u$ and $x+y=v$, evaluate $\iint_R \sin\left(\frac{x-y}{x+y}\right) dxdy$, where R is bounded by the coordinate axes and $x+y=1$ in first quadrant. **OR** Evaluate $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{(4z-x^2)}} dydxdz$	**[8]**	**CO2**
10.	Show that the vector field $\vec{F}$ given by $\vec{F}=(x^2-yz)\hat{\imath}+(y^2-zx)\hat{\jmath}+(z^2-xy)\hat{k}$ is irrotational. Find the scalar potential. **OR** Evaluate $\int_C 2xyz^2dx+(x^2z^2+z\cos yz)dy+(2x^2yz+y\cos yz)dz$ where C is any path from $(0,0,1)$ to $\left(1,\frac{\pi}{4},2\right)$.	**[8]**	**CO3**
SECTION C **(Q11 is compulsory. Q12A and Q12B have internal choices)**			
11.A	Evaluate $\iint_S \vec{A}.\hat{n}\,dS$, where $\vec{A}=(x+y^2)\hat{\imath}-2x\hat{\jmath}+2yz\hat{k}$ and S is the surface of the plane $2x+y+2z=6$ in the first octant.	**[10]**	**CO3**
11.B	Obtain the Fourier series to represent $f(x)=\frac{1}{4}(\pi-x)^2$ in the interval $0\le x\le 2\pi$.	**[10]**	**CO4**
12.A	Evaluate $\int_C [(y-\sin x)dx+\cos x\,dy]$ where C is the triangle formed by $y=0$, $x=\frac{\pi}{2}$, $y=\frac{2}{\pi}x$. **OR** Using Green's theorem, evaluate $\int_C (x^2ydx+x^2dy)$ where C is the boundary described counter clockwise of the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$.	**[10]**	**CO3**
12B.	Find the Fourier Series for the function $f(x)=x+x^2, -\pi<x<\pi$. **OR** Expand $f(x)=x$ as half range (i) *sine* series in $0<x<2$, (ii) *cosine* series in $0<x<2$.	**[10]**	**CO4**